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ON THE SANSKRUTI INDEX OF CERTAIN LINE GRAPHS OF SUBDIVISION GRAPHS

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Abstract. Derived graphs such as line, total, subdivision, r-subdivision, etc. graphs are used in making complex calculations easier. A topological index is a mathematical formula which gives some mathematical value which can be commented to get information on some property of the real life situation, a molecule, a network, etc. which is modelled by means of the graph under investigation. Recently Hosamani [7] put forward a novel topological index, namely the Sanskruti index of a molecular graph G. In this paper we compute the Sanskruti index of the line graph of tadpole, wheel and ladder graphs using the notion of subdivision.

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1 Introduction

Let G be a simple graph. The order of a graph is $|V(G)|$, its number of vertices denoted by n. The size of a graph is $|E(G)|$, its number of edges denoted by m. The degree of a vertex v, denoted by $d_G(v)$. The subdivision graph $S(G)$ is the graph attained from G by replacing each of its edges by a path of length 2. The line graph $L(G)$ of a graph is the graph derived from G in such a way that the edges in G are replaced by vertices in $L(G)$ and two vertices in $L(G)$ are connected whenever the corresponding edges in G are adjacent [6]. For any number d, we define $V_d = \{u \in V(G) \backslash S_G(u) = d\}$.

Topological indices are numerical parameters of a graph which are invariant under graph isomorphisms. Nowadays, there are many such indices that have found applications in Mathematical Chemistry especially in the quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) [1, 5, 16]. A large number of such indices depend only on vertex degree of the molecular graph. One of them is the atom-bond connectivity(ABC) index, proposed by Estrada et al. [2] and is defined as:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) d_G(v)}} \tag{1}$$

This index provides a good model for the stability of linear and branched alkanes as well as the strain energy of cycloalkanes, [2, 3]. For a collection of recent results on topological indices, we refer the interested reader to the articles [9, 8, 10, 12, 17, 18].

Inspired by work on the ABC index, Furtula et al., [4], proposed the following modified version of the ABC index and called it as augmented Zagreb index (AZI):

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3 \tag{2}$$

The prediction power is better than the ABC index in the study of heat of formation for heptanes and octanes, ([4]).

Motivated by the previous research on topological descriptors and their applications, Hosamani, [7], has put forward a novel topological index namely, Sanskruti index $\mathcal{S}(G)$ of a molecular graph G as follows:

$$\mathcal{S}(G) = \sum_{uv \in E(G)} \left(\frac{s_G(u) s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3, \tag{3}$$

in which $s_G(u) = \sum_{v \in N_G(u)} d_G(v)$ and $N_G(u) = \{v \in V(G) \backslash uv \in E(G)\}$.

2 Topological indices of $L(S(G))$

In 2011, Ranjini et. al. calculated the explicit expressions for the Shultz index of the subdivision graphs of the tadpole graph, wheel, helm and ladder graphs [14]. They also studied the Zagreb indices of the line graph of tadpole, wheel and ladder graphs with subdivision in [13]. In 2015, Su and Xu calculated the general sum-connectivity index and co-index for the line graph of tadpole, wheel and ladder graphs with subdivision, [15]. In the same year (2015), Nadeem et. al. computed the ABC_4 and GA_5 indices for the same graphs, [11].

Motivated by the results in [14, 13, 15, 11], we computed the Sanskruti index of the line graph of subdivision graph of tadpole graph, wheel graph and ladder graph respectively.

The following straightforward, previously known, auxiliary results are important for us.

Lemma 2.1. [6] *For any graph G with n vertices and m edges, the subdivision graph $S(G)$ of G is a graph with $n+m$ vertices and $2m$ edges.*

Lemma 2.2. [6] *Let G be a graph with n vertices and m edges, then the line graph $L(G)$ of G is a graph with m vertices and $\frac{1}{2}M_1(G) - m$ edges.*

3 Sanskruti index of the line graph of subdivision graph of tadpole graph

The tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of n vertices with a path of length k.

Table 1. The edge partition of G for $k = 1$

$(s_G(u), s_G(v)); uv \in E(G)$	$(3,7)$	$(5,8)$	$(7,8)$	$(8,8)$	$(4,4)$	$(4,5)$
Number of edges	1	2	2	1	$(2n-5)$	2

Table 2. The edge partition of G for $k = 2$

$(s_G(u), s_G(v)); uv \in E(G)$	$(2,3)$	$(3,5)$	$(5,8)$	$(8,8)$	$(4,4)$	$(4,5)$
Number of edges	1	1	3	3	$(2n-5)$	2

Table 3. The edge partition of G for $k > 2$

$(s_G(u), s_G(v)); uv \in E(G)$	$(2,3)$	$(3,4)$	$(4,4)$	$(4,5)$	$(5,8)$	$(8,8)$
Number of edges	1	1	$2(n+k-5)$	3	3	3

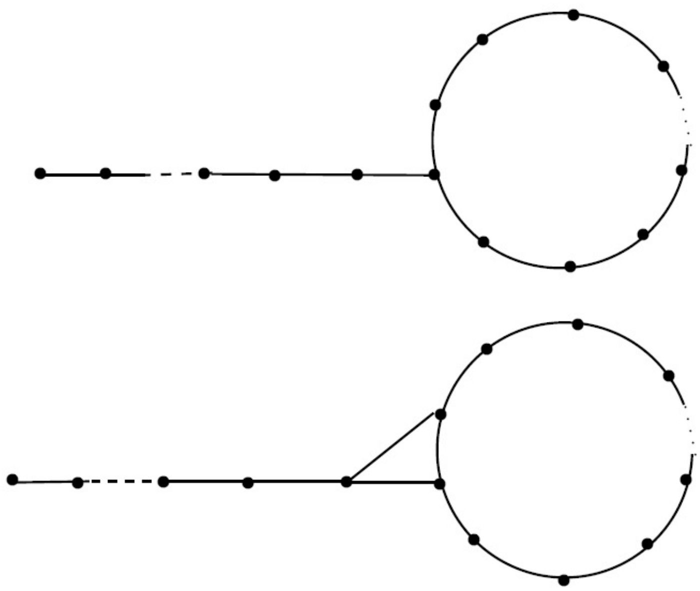

Fig.1. (a) The tadpole graph $T_{n,k}$; (b) the line graph $L(T_{n,k})$ of tadpole graph

Theorem 3.1. *Let G be the line graph of subdivision graph of the tadpole graph $T_{n,k}$, then*

$$\mathcal{S}(G) = \begin{cases} (2n-5)18.9629 + 416.3056, & \text{if } k = 1; \\ (2n-5)18.9629 + 501.1251, & \text{if } k = 2; \\ (n+k-5)37.9258 + 1454.3243, & \text{if } k > 2. \end{cases}$$

Proof. The tadpole graph $T_{n,k}$ is graph of order and size $n+k$ respectively. By Lemma2, the subdivision graph of tadpole graph $S_1(T_{n,k})$ is of order and size $2(n+k)$ respectively. Therefore, by Lemma 2, there are total $2(n+k)$ vertices in G. First consider the case $k > 2$ for G such that $|V_2| = 1$, $|V_3| = 1$, $|V_5| = 3$, $|V_8| = 3$ and $|V_4| = 2(n+k-4)$. Hence we get the edge partition based on the degree sum of neighborhood vertices of each vertex is shown in Table 3. By employing (3) to Table 3 we get the required result. By similar arguments we can obtain the expression of $\mathcal{S}(G)$ for $k = 1$ and 2 by employing (3) to Table 1 and 2 respectively.

4 Sanskruti index of the line graph of subdivision graph of wheel graph

A wheel graph W_n of order n composed of a vertex, which will be called the hub, adjacent to all vertices of a cycle of order $n-1$ i.e., $W_n = C_{n-1} + K_1$.

Table 4. The edge partition of H.

$(s_H(u), s_H(v)); uv \in E(H)$	$(9,9)$	$(9,n+6)$	$(n+6,n^2-n+3)$	(n^2-n+3,n^2-n+3)
Number of edges	$2n$	$2n$	n	$\frac{n(n-1)}{2}$

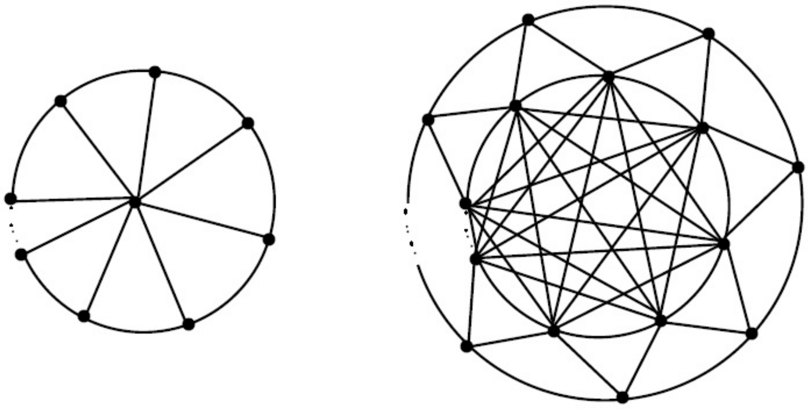

Fig.2. The Wheel graph W_n and its line graph $L(W_n)$

Theorem 4.1 *Let H be the line graph of subdivision graph of the wheel graph W_n, then*

$$\mathcal{S}(H) = 2n\left(\frac{9(n+6)^3}{(n+13)^3}\right) + n\left(\frac{[(n+6)(n^2-n+3)]^3}{(n^2+7)^3}\right) + \frac{n(n-1)}{2}\left(\frac{(n^2-n+3)^5}{2(n^2-n+2)^3}\right) + (259.4926)n$$

Proof. Employ (3) to the edge partition shown in Table 4 to get the required result.

4.1 Sanskruti index of the line graph of subdivision graph of ladder graph

A ladder L_n is obtain by taking cartesian product of two paths $P_n \times P_2$.

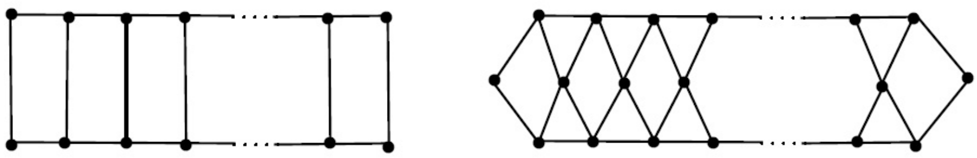

Fig. 3. The ladder graph $P_n \times P_2$ and its line graph $L(P_n \times P_2)$

Table 5. The edge partition of K for $n = 3$

$(s_K(u), s_K(v)); uv \in E(K)$	$(4,4)$	$(4,5)$	$(5,8)$	$(8,8)$	$(8,9)$	$(9,9)$
Number of edges	2	4	4	2	4	1

Table 6. The edge partition of K for $n > 3$

$(s_K(u), s_K(v)); uv \in E(K)$	$(4,4)$	$(4,5)$	$(5,8)$	$(8,9)$	$(9,9)$
Number of edges	2	4	4	8	$(9n-28)$

Theorem 5.1. *Let G be the line graph of subdivision graph of the tadpole graph $T_{n,k}$, then*

$$\mathcal{S}(K) = \begin{cases} 1086.7381, & \text{if } n = 3; \\ (9n-28)129.7463 + 1208.2928, & \text{if } n > 3. \end{cases}$$

Proof. The line graph of subdivision graph of ladder graph contains $6n-4$ vertices. Consider the case for $n > 3$ such that $|V_4| = 4$, $|V_5| = 4$ and $|V_8| = 4$ and $|V_9| = 2(3n-8)$. Hence we get the dedge partition based on the degree sum of neighbor vertices of each vertex as shown in Table 6. By employing (3) to Table 6, we get the required result. Similar argument will work for the case $n = 3$.

Summary and Conclusion In chemical graph theory, there is a new direction in the field of structural chemistry under the framework of subdivison graph operator. Motivated by the fact that the derived graphs such as line, total, subdivision, r-subdivision graphs are being frequently used in making complex calculations easier, the authors computed the Sanskruti index of the line graph of tadpole, wheel and ladder graphs by means of the notion of subdivision. This is a new usage of topological indices which are simple mathematical formulae which give some mathematical values which can be commented to get information on some properties of the real life situation, a molecule, a network, etc. which are modelled by means of the graph under investigation.

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