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PRODUCTION PLANNING OF A MANUFACTURING INDUSTRY USING LINEAR PROGRAMMING

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Abstract. Considering the intensification of technology in the 21^{st} century, studies of optimization models have been necessary for the control and planning of productive systems. In this scenario, the goal of this paper is to minimize the production costs of a manufactory by applying the Combined Problem, which couples the lot sizing and stock cutting problems. Numerical data are collected from a small-scale furniture plant, located in Cornélio Procópio city, in the state of Paraná, Brazil. Two production planning scenarios are considered: first, costs and demands of the final products are constants; second, costs and demands vary over periods of planning. Case study involving larger data is simulated. Numerical solutions are obtained from the application of the Simplex method with computational support and indicates the quantity to be produced in each planning period. These results are compared with the case experienced by the manufactory, whose production is made considering the demand by period. The production planning obtained by linear programming provides support to the decision make of the manufactory.

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1 Introduction

Due to technological and computational advances, manufacturing industries have been encouraged to make their processes more efficient and competitive, while minimizing global production costs. Reducing expensive raw material waste is an important goal in the industry. Thus, the study of optimization models for the control and production planning becomes an essential tool for industrial advancement, motivating academic researches.

In this scenario, the Operational Research has great importance and can be defined as a science that solves problems quantitatively, using mathematical and statistical methods, and it helps making the best decisions. The scientific component is related to the mathematical modeling of decision problems, determining the objectives and constraints under which to operate [2].

Production management within an industry is responsible for planning and controlling the transformation of raw materials into final products. The system responsible for this management is called Production Planning and Control (PPC) [4], which coordinates the activities, from the acquisition of raw materials to the delivery of the final products. This way, the Operational Research and its optimization methods have great utility in solving problems, especially those involving productive processes, decision making and systems management, selecting the best decisions, among all possible ones [9].

Usually, industries produce pieces of different sizes and materials in order to construct a final product. In this process, industries worry about the waste of raw materials, since this implies a profit reduction. Then, the question is to solve an optimization problem, which consists of cutting objects, observing these issues.

In [5], two robust mixed integer programming models are proposed. In the first one, the non-overlapping constraints are stated based on direct trigonometry and in the second one, pieces are decomposed into convex parts and then the non-overlapping constraints are written based on nofit polygons of the convex parts. Two-dimensional irregular strip packing problems consist of cutting and packing problems in which an object must be cut in small pieces, involving a non-trivial handling of geometry. Computational experiments with benchmark instances show that second model outperforms the first one and also the best exact model published in the literature.

In this sense, in [11], two-dimensional irregular cutting stock problem with demand is studied, in which the required pieces has to be produced from large rectangular sheet minimizing material waste. Greedy randomized adaptive search procedure (GRASP) metaheuristic algorithm is adapted to tackle the problem by providing high-quality solution in an appropriate time.

About three-dimensional cutting problems, in [14] the cutting problem of a marble processing factory is described. In order to minimize total spoilage of marble while finding the cutting designs of marble blocks and marble planes, integer programming approaches were developed and satisfactory results were reached.

In [1], an integrated production planning and cutting stock problem is investigated. This kind of problem appears frequently in small-scale furniture plants. The production costs contained in the manufacturing process and the products demands are not known. Since there are uncertainties, robust optimization models were proposed. These models control the conservatism of the solution according to the attitude of the decision maker towards risk.

The purpose of [6] is to solve the problem of optimizing the production on the basis of the commercial value of the cuts, since the loss of raw materials in wood cutting industries has reached high. This work was completed with the design and the presentation of a software package called cutting optimizer.

The study of [15] deals with a case in which there are several stock lengths available in limited quantities. The heuristic methods are empirically analyzed by solving a set of randomly generated instances and a set of instances from the literature. The proposed methods presented very small objective function value gaps.

Fast heuristics, Integer Linear Programming models and a truncated Branch-and-Price algorithm for a wooden board Cutting Stock Problem are proposed in [10]. They consider the maximization of the cutting equipment productivity, which can be obtained by cutting identical boards in parallel. Experiments show that the productivity can be improved with a minimal increase in the total area of used boards.

In the context of stock cutting and lot sizing problems, the goal of this work is to propose the application of a mathematical model to a real case of a manufactory, in order to minimize the global production costs. Real data are collected from a small-scale furniture plant, located in Cornélio Procópio city, in the state of Paraná, Brazil. Costs and demands vary over periods of planning, which represents a more realistic situation. Two production planning scenarios are considered: costs and demands of the final products constants and costs and demands vary over periods of planning. Case study involving big data is simulated. Numerical results from this study are compared with the case whose production is made considering the demand by period. The main contribution is to propose an efficient production planning, using linear programming, providing support to the decision make of the manufacturing industry.

2 The Combined Problem

In the process of cutting a board into smaller pieces, for producing items, material loss tends to be increasingly reduced if the cuts are rearranged in a convenient way on the board. Due to this fact, there is an economic pressure to manufacture some products in advance in order to minimize losses. However, this stock can generate costs that can make the production slower [3]. This decision problem is called *Combined Problem*, which couples two optimization problems: the lot sizing and cutting stock [8].

The lot sizing problem is to plan the amount of items to be produced in various stages in each period over a horizon of finite time, in order to meet demand and to optimize an objective function, as minimizing production and storage costs[2]. It can be classified as mono-stage, where items are independently produced, and multistage, where productions of items are dependent.

The cutting stock problem consists in optimizing the process of cutting boards into smaller pieces in quantities and sizes demanded. Cutting pattern is defined as the arrangement of the pieces within each board, that is, the way an object (piece) is cut to produce demanded items. Some rules are defined, such as guillotine cuts (where each cut made on a rectangular board produces two new rectangles), limiting pieces (restricted or unrestricted cuts), number of stages (2-stages is when Only one change in the direction of guillotine cuts is allowed: horizontal / vertical or vertical / horizontal). In addition, the problem will be two-dimensional when two dimensions are relevant for cutting.

Thus, the goal of the combined problem is to decide the number of final products to be produced in each planning horizon period in order to minimize production, preparation and storage costs (lot sizing) and the number of boards to be cut, as well as cutting patterns to compose final products (cutting stock) [8]. In real situations, most industries approach these two problems separately, which can raise global costs.

One approach to the combined problem disregards preparation costs and relaxes the integrality of the variables representing the number of boards cut in a given pattern, which requires a large demand. This approach can be applied in the furniture industry where wooden boards must be cut to produce items. It is considered that there is only one type of board and parameters like costs and demands are varied [3].

Indexes:
t=*1, ...,* T number of periods of time.
p=*1, ...,* P number of different pieces to be cut.
j=*1, ...,* N number of different cutting pattern.
i=*1, ...,* M number of different final products.

Parameters:
c_{it}: production cost of the final product i in the period t.
h_{it}: stocking cost of the final product i in the period t.
hp_{pt}: stocking cost of the piece p in the period t.
d_{it}: demand of the final product i in the period t.
r_{pi}: number of pieces p required to make product i.
v_j: time to cut a board using cutting pattern j.
a_{pj}: number of piece p cut using cutting pattern j.
u_t: maximum time of saw capacity.
cp: cost of the board to be cut.

Decision Variables:
x_{it}: number of final product i produced in the period t.
e_{it}: number of final product i stocked in the end of the period t.
ep_{pt}: number of piece p stocked in the end of the period t.
y_{jt}: number of boards cut using cutting pattern j in the period t.
The mathematical model is formulated according to [3] and it is described as equations (1) to (5).

$$\min \sum_{i=1}^{M}\sum_{t=1}^{T}(c_{it}x_{it} + h_{it}e_{it}) + \sum_{j=1}^{N}\sum_{t=1}^{T} cpy_{jt} + \sum_{p=1}^{P}\sum_{t=1}^{T} hp_{pt}ep_{pt} \tag{1}$$

subject to:

$$x_{it} + e_{i,t-1} - e_{it} = d_{it} \tag{2}$$

$$\sum_{j=1}^{N} a_{pj} y_{jt} + ep_{p,t-1} - ep_{pt} = \sum_{i=1}^{M} r_{pi} x_{it} \quad \forall \ t = 1 \dots T \tag{3}$$

$$\sum_{j=1}^{N} v_j y_{jt} \leq u_t \tag{4}$$

$$x_{it}, e_{it}, y_{jt}, ep_{pt} \geq 0 \tag{5}$$

The restrictions (2) are related to the stock balance equations in relation to final products, which ensures that the demand for items from each period will be met. The restrictions (3) are related to stock balance equations in relation to pieces, which ensures that the demand for pieces will be satisfied. These restrictions are those that couple the lot sizing and stock cutting problems since both include the x_{it} variables, which define the lots size and y_{jt}, which define the number of boards cut in a certain cutting pattern. The restrictions (4) are related to saw capacity, which ensures that the time spent in the cutting process of the boards in the various cutting patterns does not exceed the available capacity of the saw, that is, its maximum operating time. And, finally, (5) represents non-negativity conditions.

The application of Simplex Method [2, 9] with columns generation [7] has been presented, in the literature, as a very efficient strategy to solve this kind of linear problem.

3 Simplex Method

The Simplex Method, applied to provide the solution of Linear Programming Problems, can be used to solve real situations that involve lot sizing and cutting stock problems [2, 9].

The Simplex Method, developed in 1947 by the North American mathematician George B. Dantzig, consists of an iterative numerical procedure, which executes repeatedly a sequence of steps, in order to reach the best solution of the problem, called *optimal solution*, if it exists. The procedure starts from a viable basic solution, belonging to a vertex, of the system of equations that represents the constraints of the problem. From this initial solution, the algorithm identifies new viable solutions of equal or better value than current. Thus, the process finds new vertices of the convex envelope of the problem and determines if this vertex is optimal or not, that is, if the change of variables in the base can still improve the objective function. Since the Simplex Method is an iterative process, can be implemented using any programming language to execute its iterations [3].

The algorithm of the Primal Simplex Method is described below.

phase I

Find a primal-feasible basic partition: $A = (B, N)$.
Do STOP=FALSE, IT=0
(It will be FALSE until the optimality condition is verified. IT indicates the iteration number.)

phase II

While NOT STOP do:

- Determine the basic feasible primal solution: $x_B = B^{-1}b$.

- Optimality test:
  Determine the dual basic solution: $y^T = c_B^T B^{-1}$;
  Find x_k with relative cost: $c_k - y^T a_k < 0$.
  If $c_k - y^T a_k \geq 0, \quad \forall\ k = 1, \ldots, n-m$, then the IT iteration solution is optimal.

  STOP=TRUE.
  Else:

- Find the simple direction: $d_B = -B^{-1}a_k$, of change in the values of the basic variables.

- Determine the step: $\epsilon^0 = min \left\{ -\frac{x^0_{B_e}}{d_{B_e}} | d_{B_e} < 0, i = 1, \ldots, m \right\}$.
  If $d_B \geq 0$, the problem does not have optimal finite solution.
  STOP=TRUE.
  else:

- Update the basic partition: $a_{B_l} \leftrightarrow a_k, IT \leftarrow IT + 1$.

  end while.

4 Case Study

In order to execute the Combined Problem, values were assigned to its parameters derived from data provided by a small-scale furniture plant, located in the city of Cornélio Procópio, in the state of Paraná, Brazil. So that it was possible to decide on two productions scenarios described below. Both consider producing two types of end products: tables and chairs. In addition, in both cases, it is considered that there is no stock in the previous period $t-1$. For each scenario, it is firstly assumed that costs and demand values are constant throughout the production planning periods and, subsequently, the variation of these parameters over the periods.

4.1 The First Production Planning Scenario

Considering costs and the demand for final products *constants*, the following data is available in the first production planning scenario:
$t = 6$ time periods
$p = 3$ pieces
$j = 5$ types of board cutting patterns
$i = 2$ final product (table and chair)

The pieces for the composition of the final products are:
Piece 1: cover
Piece 2: foot
Piece 3: seat/backrest

The variable x_{1t} represents the product "table", whose production cost is c_{1t}=R\$255 and the demand is d_{1t}=2 for $t = 1, ..., 6$. The variable x_{2t} represents the product "chair", whose production cost is c_{2t}=R\$80 and the demand is d_{2t}=3 for $t = 1, ..., 6$. The number of pieces p required to make final product i can be seen in Table 1. The other parameters provided from the manufactory are:
$cp = R\$120$,
$u_t = 300$ hours per period,
a_{pj} can be seen in Table 2,
$r_{11} = 1$, $r_{21} = 5$, $r_{32} = 2$, $r_{22} = 6$,
$h_{1t} = 3$, $h_{2t} = 1$,
$hp_{1t} = 0,2$, $hp_{2t} = 0,3$, $hp_{3t} = 0,5$.

Table 1: Number of pieces p required to make product i

Piece	**Table**	**Chair**
$p = 1$	1	0
$p = 2$	5	6
$p = 3$	0	2

Table 2: Cutting Patterns: first production planning scenario

Pattern	**Piece 1**	**Piece 2**	**Piece 3**	**Time**
j=1	2	0	0	v_1=1s
j=2	1	88	0	v_2=1.2s
j=3	0	0	35	v_3=1.5s
j=4	0	0	45	v_4=1.4s
j=5	1	8	15	v_5=1.5s

The pieces to be cut are cover ($p = 1$), foot ($p = 2$) and seat/backrest ($p = 3$). The cutting patterns shown in Table 2 are pre-set by the manufactory, according to the capacity of the equipment and the available labor.

The second production planning considers that costs and demands are *variables* throughout the planning periods. The new parameters provided by the manufactory under study are described below. The production costs of final products can be seen in Table 3, which corresponds to parameters c_{it}.

The cost of the board to be cut is $cp = 135.07$. The stocking costs of the final products are shown in Table 4 (parameters h_{it}) and the stocking costs of the pieces are shown in Table 5 (parameters hp_{pt}).

Table 6 shows the demands (d_{it}) of the final products.

Table 3: Production costs of the final product i in the period t

Time period / **Final product**	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Table	R$ 255	R$ 255	R$ 255	R$ 267.75	R$ 267.75	R$ 267.75
Chair	R$ 80	R$ 80	R$ 80	R$ 84	R$ 84	R$ 84

Table 4: Stocking cost of the final product i in the period t

Time period / **Final product**	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Table	R$ 3	R$ 3	R$ 5	R$ 5	R$ 5	R$ 5
Chair	R$ 1	R$ 1	R$ 3	R$ 3	R$ 3	R$ 3

Table 5: Stocking cost of the piece p in the period t

Time period / **Pieces**	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
$p = 1$	R$ 0.20	R$ 0.20	R$ 0.50	R$ 0.50	R$ 0.50	R$ 0.50
$p = 2$	R$ 0.30	R$ 0.30	R$ 0.60	R$ 0.60	R$ 0.60	R$ 0.60
$p = 3$	R$ 0.50	R$ 0.50	R$ 0.80	R$ 0.80	R$ 0.80	R$ 0.80

The optimal solution of these two problems, following the application of the algorithm Simplex, should indicate in which period of the planning horizon and in what quantity the final products must be produced, so as to obtain the minimum cost of cutting and stock, respecting the stock balance restriction to final products and boards, the capacity constraint of the saw and the non-negativity conditions.

Table 6: Demand of the final product i in the period t

Time period / **Final product**	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Table	2	3	2	4	1	3
Chair	3	5	4	6	3	4

The design of table and chair (final products) produced in this scenario is illustrated in Figure 1.

Figure 1: Design of table and chair

4.2 The Second Production Planning Scenario

Considering costs and the demand for final products **constants**, the following data are obtained in the second production planning scenario:
$t = 6$ time periods
$p = 7$ pieces
$j = 6$ types of board cutting patterns
$i = 2$ final product (table and chair)

The pieces for the composition of the final products are:
Piece 1: cover
Piece 2: backrest
Piece 3: seat
Piece 4: chair support
Piece 5: foot of the chair
Piece 6: table support
Piece 7: foot of the table

In this case, the new cutting patterns for the production of parts are described in Table 7. These are pre-set by the manufactory, according to the capacity of the equipment and the available labor.

Table 7: Cutting Patterns: second production planning scenario

Pattern	p=1	p=2	p=3	p=4	p=5	p=6	p=7	Time
j=1	0	34	34	0	0	0	0	v_1=3s
j=2	15	8	7	0	0	0	0	v_2=2s
j=3	12	12	13	0	0	0	0	v_3=4s
j=4	0	0	0	8	1	2	0	v_4=4s
j=5	0	0	0	2	3	4	0	v_5=3s
j=6	0	0	0	0	0	0	4	v_6=2s

The variable x_{1t} represents the product "table", whose production cost is R$60 and demand is 10, and the variable x_{2t} represents "chair", whose production cost is R$40 and demand is 20. The parameters (production cost, stock and cut time) were changed, since data from other types of wood and other cutting patterns were used. It is also considered that there is no stock in the previous period $t-1$.

Parameters provided by the manufactory:
$cp = R\$135.07$
$u_t = 240$ hours per period
The number of pieces p required to make product i, which corresponds to parameters r_{pi}, can be seen in Table 8.

Table 8: Number of pieces p required to make product i

Piece	**Table**	**Chair**
$p = 1$	1	0
$p = 2$	0	1
$p = 3$	3	1
$p = 4$	0	2
$p = 5$	0	4
$p = 6$	2	0
$p = 7$	4	0

The second production planning, considers that costs and demands are *variables* throughout the planning periods. The new parameters provided from the manufactory under study are described below. The costs of the final products, c_{it}, are provided in Table 9.

The cost of the board to be cut is $cp = 135.07$.
The stocking costs of the final products, h_{it}, can be seen in Table 10 and the stocking costs of the pieces, hp_{pt}, are provided in Table 11.
Demands of the final products are given in Table 12.

Table 9: Production cost of the final product i in the period t

Time period / Final product	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Table	R$60	R$60	R$60	R$75	R$75	R$78
Chair	R$40	R$40	R$42	R$45	R$45	R$50

Table 10: Stocking cost of the final product i in the period t

Time period / Final product	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Table	R$4	R$4	R$4	R$5	R$5	R$5
Chair	R$2	R$2	R$3	R$3	R$4	R$5

After running the new combined model using algorithm Simplex, the optimal solution will indicate in which period of the planning horizon and in what quantity the final products must be produced in these two problems, in order to obtain the minimum cost, respecting restrictions (2) to (5) of the Combined Problem.

4.3 Simulations

Some simulations are executed in order to exemplify larger production scenarios. For that, two types of furniture have been added in final products: stool and wardrobe. Costs and demands parameters were considered as constants over the time periods of production planning, since the manufactory data history shows that the value variation is not relevant for the last six months.

Simulating the first production planning scenario, described in Section 4.1, the following data is obtained from the manufactory history:
$t = 6$ time periods (maximum planning of the manufactory)
$p = 3$ pieces
$j = 5$ types of board cutting patterns
$i = 4$ final product (table, chair, stool and wardrobe)

The pieces used to compose the final products are:
Piece 1: cover
Piece 2: foot
Piece 3: seat/backrest

The variable x_{1t} represents the product "table", whose production cost is c_{1t}=R$255 and the demand is d_{1t}=2 for $t = 1, ..., 6$. The variable x_{2t} represents the product "chair", whose production cost is c_{2t}=R$80 and the demand is d_{2t}=3 for $t = 1, ..., 6$. The variable

Table 11: Stocking cost of the piece p in the period t

Time period / Pieces	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
$p=1$	R$ 0.40	R$ 0.40	R$ 0.40	R$ 0.50	R$ 0.50	R$ 0.50
$p=2$	R$ 0.35	R$ 0.35	R$ 0.40	R$ 0.50	R$ 0.50	R$ 0.60
$p=3$	R$ 0.25	R$ 0.25	R$ 0.30	R$ 0.30	R$ 0.50	R$ 0.70
$p=4$	R$ 0.13	R$ 0.13	R$ 0.15	R$ 0.20	R$ 0.20	R$ 0.30
$p=5$	R$ 0.15	R$ 0.15	R$ 0.20	R$ 0.30	R$ 0.30	R$ 0.50
$p=6$	R$ 0.18	R$ 0.18	R$ 0.18	R$ 0.25	R$ 0.25	R$ 0.25
$p=7$	R$ 0.23	R$ 0.23	R$ 0.23	R$ 0.30	R$ 0.30	R$ 0.30

Table 12: Demand of the final product i in the period t

Time period / Final product	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
Table	10	6	8	14	12	13
Chair	20	16	12	24	14	18

x_{3t} represents the product "stool", whose production cost is c_{3t}=R$50 and the demand is d_{3t}=6 for $t = 1, ..., 6$. Finally, the variable x_{4t} represents the product "wardrobe", whose production cost is c_{4t}=R$1440 and the demand is d_{4t}=2 for $t = 1, ..., 6$.

The other parameters provided from the manufactory are:
$cp = R\$120$,
$u_t = 300$ hours per period,
a_{pj} the same as Table 2,
$h_{1t} = 3$, $h_{2t} = 1$, $h_{3t} = 1$, $h_{4t} = 5$
$hp_{1t} = 0.2$, $hp_{2t} = 0.3$, $hp_{3t} = 0.5$.

The number of pieces p required to make product i, r_{pi}, is given in Table 13.

Table 13: Number of pieces p required to make product i

Piece	**Table**	**Chair**	**Stool**	**Wardrobe**
$p=1$	1	0	0	8
$p=2$	5	6	4	0
$p=3$	0	2	1	0

Simulating the second production planning scenario, described in Section 4.2, and considering costs and demands for final products constants, the following data is obtained from the manufactory history:

$t = 6$ time periods
$p = 7$ pieces
$j = 6$ types of board cutting patterns
$i = 4$ final product (table, chair, stool and wardrobe)

The pieces required to compose the final products are:
Piece 1: cover
Piece 2: backrest
Piece 3: seat
Piece 4: chair support
Piece 5: foot of the chair
Piece 6: table support
Piece 7: foot of the table

The variable x_{1t} represents the product "table", whose production cost is R$60 and demand is 10 per period, and the variable x_{2t} represents "chair", whose production cost is R$40 and demand is 20. The variable x_{3t} represents the product "stool", whose production cost is R$30 and demand is 12. And the variable x_{4t} represents the product "wardrobe", whose production cost is R$240 and demand is 5 per period.

Other parameters provided by the history manufactory are:
cp= R$135.07,
$u_t = 240$ hours per period,
a_{pj}: the same as Table 7.

The number of pieces p required to make product i, r_{pi}, is given in Table 14.

Table 14: Number of pieces p required to make product i

Piece	**Table**	**Chair**	**Stool**	**Wardrobe**
$p = 1$	1	0	0	14
$p = 2$	0	1	0	0
$p = 3$	3	1	1	0
$p = 4$	0	2	4	0
$p = 5$	0	4	0	0
$p = 6$	2	0	0	0
$p = 7$	4	0	0	0

4.4 Numerical Results

Optimum solutions were obtained from the execution of the previously described models with computational support of the LINDO software (Linear Interactive and Discrete Optimizer) from the execution of algorithm Simplex.

For the first production planning scenario, considering the costs and demand of final products *constants*, the optimal solution obtained then suggests anticipating the production of chairs, $x_{21} = 18$, generating stock, postponing table production, $x_{13} = 12$. Thus, the optimal solution yields savings 15.2% of profit for each period, when it is compared with the production cost that satisfies the demand in each time period (without optimization).

Still in the first production scenario, but now considering a *variation* in costs and demand, the optimal solution obtained suggests to anticipate the production of chairs $x_{21} = 25$ and postpone the production of tables, $x_{13} = 15$. Compared to the production cost that satisfies the demand in each time period, this optimal solution provides a total savings 29.7% of profit.

For the second production planning scenario, considering costs and demand for final products *constants*, the solution suggests the production of chairs in the third period $x_{23} = 120$ and the production of tables in the fifth period $x_{15} = 60$. Comparing to the cost of non-optimized production, this optimal solution provides a total savings 3.9% of profit.

Considering the *variation* of costs and demand in the second production planning scenario, the optimal solution obtained suggests the production of tables $x_{12} = 63$ and chairs $x_{22} = 104$, both in the second period. This solution provides a total savings 17.13% of profit, when compared to the production cost that satisfies the demand in each time period.

Simulating larger production scenarios, considering the first production planning scenario and costs and demands of final products *constants* (according to the manufactory data history), the optimal solution suggests anticipating for first period the production of tables and stools, $x_{11} = 12$ e $x_{31} = 36$, generating stock. In addiction, it suggests to produce chairs in the second period, $x_{22} = 18$, and postponing wardrobe production in the sixth period, $x_{46} = 12$. Thus, the optimal solution saves 22.04% of profit for each period, when it is compared with the production cost that satisfies the demand in each time period.

Simulating the second production planning scenario, the optimal solution suggests anticipating for first period the production of tables and some of chairs, $x_{11} = 60$ e $x_{21} = 37.53$, generating stock. It also suggests to produce stools in the second period, $x_{32} = 72$, and postponing wardrobe and the rest of chairs production in the sixth period, $x_{46} = 30$ e $x_{26} = 82.46$. This optimal solution saves 27.93% of profit for each period, when it is compared with the production cost that satisfies the demand in each time period.

For the solutions comparison purposes, the *Branch-and-bound* [13] algorithm was applied in all considered scenarios of production. This algorithm consists in dividing the problem into smaller sub-problems, until those can be solved using just entire decision variables [9, 2]. Since *Branch and Bound* aims to obtain just integer solutions, for all scenarios, except for the last simulation, entire optimal solutions are the same as the ones obtained from Simplex. For the last simulation the results are: $x_{11} = 60, x_{21} = 38, x_{46} = 30, x_{32} = 72$ and $x_{26} = 82$. Thus, both algorithms are efficient to provide the best solutions of considered linear programming problems. They may differ in the number of iterations performed to obtain the optimal solution, as informed in Table 15. In this table, "scenario1-const" corresponds to the first production planning scenario, described in Section 4.1, considering the parameters constants and "scenario1-var" considering the variation of the parameters. In addiction, "scenario2-const" corresponds to the second production planning scenario, described in Section 4.2, considering the parameters constants and “scenario2-var” considering the variation of the parameters. Still in the table, “simulation 1” is the simulation of the first production planning scenario and “simulation 2” corresponds to the second one.

Table 15: Number of iterations

Model	**Simplex**	**Branch and Bound**
scenario1-const	4	7
scenario1-var	4	7
scenario2-const	3	4
scenario2-var	3	4
simulation 1	5	6
simulation 2	6	15

5 Conclusion

This paper presented a study case in two programming scenarios of Combined Problem production and two simulations, which involves two important problems of linear optimization: *lot sizing* and *cutting stock*.

Treating them separately can raise global production costs, especially if a significant portion of the cost of the final product is formed by the material to be cut. Despite the fact that the combination of these problems is still little explored in the literature, the relevance of this combination in different situations elects this problem as an important issue to be researched.

In order to obtain the optimal solution in both production planning scenarios, the Simplex Method was applied, knowing that in the literature this method, together with the generation of columns, is the most recommended for the solution of the Combined Problem.

The optimal solutions to the problems were analyzed in both production planning scenarios, considering constant the costs and demand of final products and, subsequently, the variations of such costs and demand, for 6 periods. In both cases it was possible to observe the optimization, through the profit obtained using the Combined Problem in conjunction with the Simplex Method, when compared to the production cost that satisfies the demand in each time period. Larger production scenarios were considered and simulated, illustrating the real situation experienced by the manufactory. The obtained results from the simulations indicate similar behavior of numerical solutions, which provide profits and avoids the waste of raw material.

In practical situations, a strategy can be applied to round fractional variables to assume integer values, in order to obtain a viable solution [12]. Because the number of boards in a solution must be integer, an optimal solution must use at least one board. Thus, if the solution obtained by rounding is not optimal, at most one more board is used than the optimal solution.

As future works, we intend to build a computational interface so that users can obtain the optimal solution easily, just inserting the known parameters of the problem, even without understanding the mathematical model.

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