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THE CLASS/TEACHER ASSIGNMENT PROBLEM: A CASE STUDY IN A BRAZILIAN UNIVERSITY

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Abstract. In this work, a mathematical model to assign classes to professors is proposed, aiming at a maximization of a convex function depending on their preferences and skills. The mathematical model consists of an integer binary linear program, whose objective allows for both satisfaction and productivity optimizations, which is considered a more rational use of their skills. The method applied was a case study exploratory research. The data used to assess the goodness of the mathematical model were collected on the Mathematics Department to which the first author are affiliated.

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1 Introduction

University timetabling represents a difficult optimization problem and finding a high quality timetable is a challenging task. With a large number of events involved and various hard constraints to be considered, finding an optimal timetable is time consuming [1]. According to [14], timetabling problems are a specific type of scheduling problem and are mainly concerned with the assignment of events to timeslots subject to constraints with the resultant solution constituting a timetable.

The courses given at undergraduate level in Brazilian universities are mostly designed for one semester or one academic year periods. The subjects to be offered are associated with a timetable and group of professors, usually in a complex way. In this paper, we start from the assumption that the timetabling is given, and we set a integer binary linear program to make the distributions of subjects to each of the professors, according to their skills and availabilities. The contracts weekly hours also vary, ranging from 8 to 16 hours in classes. The problem may be stated as follows: how to perform the distribution efficiently, meeting the team expectations?

Some working hypotheses will be designed. First of all, every subject must be assigned to at least one professor, but, in some cases, a subject may be assigned to up to three professors.

It is also common that some professors are lecturing in more than one institution, therefore needing some time to commute form one place to another. Further, some professors make requirements not to be assigned to certain times due to other pre-scheduled appointments.

Some professors may have prerogatives over subjects and timetable, as well as there are preferences among courses. Professors preferences may be also associated with a class size and shifts (usually the night shift has less preference).

Some departments give privileges to more experienced professors, with active research or with higher degrees, but an egalitarian approach can also be followed.

When considered altogether, or partly, these conditions lead, more frequently than not, to some gaps in the timetable assignment. We propose to attack this problem from the Operations Research perspective.

We shall use as case study the Maths Department of the State University of Londrina (Universidade Estadual de Londrina – UEL – PR – Brazil), which has a fairly complex assignment problem. Presently, the department has to serve around 10 undergraduate and 5 graduate courses. About 10 of its 40 professors are temporary, mostly having other duties besides the contract with the University. The model we devise seek to encompass on top of that some restrictions in abilities to lecture very specific subjects. This present study was motivated by real experimented situations on this cited Mathematics Department, since professors have preferences to develop a specific activity or subject, but they do not have the required skill for that. From these detected situations, the idea is to propose a mathematical model that consider not only to maximize the professors satisfaction, but also their skills. Thus, this model must be able to harmonize satisfaction and efficiency of the team, by considering the skills of the professors. In this sense, the contribution of this paper is to propose an alternative model which is able to consider the satisfaction of professors and to assign tasks so that the team be more efficient.

Briefly, the mathematical model is based on a integer linear program, adjusting some parameters according to the conditions we described. We simulated various assignment scenarios which may be applied in our case study and adapted to others.

In Section 2 we present a brief review of the literature on the subject. In Section 3, we state the mathematical model and explain its parameters. In Section 4 we describe graphical results of the simulations and end up with our conclusions in Section 5.

2 Literature review

There are several approaches in the assignment problem, concerning the various aspects, as professor, class and rooms. In recent years, interest in meta-heuristic approaches such as Simulated Annealing, Tabu Search and Genetic Algorithms, for university timetabling, has increased since these approaches generate better solutions than those generated from sequential heuristics alone. We briefly review some of these approaches.

Colorni, Dorigo and Maniezzo [5] uses the technique of *Simulated Annealing*, with Tabu Search and Genetic Algorithms, in the assignment of teachers in Italian schools, concluding that the first technique had the worst performance. The authors compared the simulated results with made manually assignments. Cowling et. al. [6] also investigated a genetic algorithm based hyperheuristic (hyper-GA) for scheduling geographically distributed training staff and courses. The aim of the hyper-GA was to evolve a good-quality heuristic for each given instance of the problem and use this to find a solution by applying a suitable ordering from a set of low-level heuristics.

Carter and Laporte [4] have developed a computerized examination timetabling system called *examine*. Several algorithmic strategies for this problem are investigated and compared. Results are reported on some real problems. Considering the examination timetabling problem, Nuitjen et al. [11] applied a general constraint satisfaction technique to this problem at the Eindhoven University of Technology.

An approach to the student-scheduling problem is presented by [7]. Students satisfaction with regard to individually specified preferences for various aspects of the scheduling are used as the objective which yields a multi-criteria decision problem. The resulting mixed-integer programme is then modeled.

Kingston [9] built an object oriented software, based on the internet, known as KTS. The author developed it working with Australian teaching data and claimed his software is very efficient.

About applying hybrid approaches, White and Zhang [12] aim that the sequential use of a constraint logic program whose output was used to start the Tabu Search produced the best timetables of all in a time that was much longer than that of the logic program alone but shorter than that of the Tabu Search used alone. Merlot et al. [10] implemented a hybrid algorithm for examination timetabling, consisting of three phases: a constraint programming phase to develop an initial solution, a simulated annealing phase to improve the quality of solution, and a hill climbing phase for further improvement. The hybrid method was compared to established methods on the publicly available data sets, and found to perform well in comparison.

Burke *et al.* [3] present an investigation of a simple generic hyper-heuristic approach upon a set of widely used graph coloring heuristics in timetabling. A Tabu Search approach is employed to search for permutations of graph heuristics which are used for constructing timetables in exam and course timetabling problems. The approach represents a significantly more generally applicable approach than the current literature.

Avella *et al.* [2] proposed a two phase method. Firstly the use Simulated Annealing, and on the second phase, they made a local search, using random descent. The authors based their work on the method of Very Large-Scale Neighborhood to solve timetable problem.

The research [1] aims to build upon the state of the art in search methodologies for university timetabling. It focuses on both examination and course timetabling problems. The research first highlights an initial investigation into a very large scale neighbourhood search for examination timetabling problem. Computational results based on standard university benchmark instances are reported to demonstrate the effectiveness of the approaches studied.

Wilke and Ostler [13] used Branch-and-Bound method, Tabu Search, Simulated Annealing and Genetic Algorithm, reporting that the best solutions were obtained with Tabu Search.

Irene, Deris and Hashim [8] combined *Particle Swarm Optimization* (PSO) with local search techniques to solve the timetable problem. The authors compared the performance of PSO solution with their combined method, concluding that the combined approach provides better solutions.

The extensive diversity of approaches to the timetable problem stands for its relevance and current interest. We will next show our proposed formulation of a binary linear program model for the timetable problem designed from actual offered courses data of the Mathematics Department at University of Londrina.

3 Proposed Mathematical Model

3.1 Description of the DMAT/UEL Scenario

In DMAT/UEL there are 40 professors and about 130 subjects to be assigned whose schedules are pre-defined. Consequently, some auxiliary sets are considered, identified as I, J, T and K. The first one, $I = \{1, 2, \cdots, m\}$, represents professors, the second one, $J = \{1, 2, \cdots, n\}$, represents subjects. The third set, $T = \{1, 2, \cdots, 150\}$, identifies the lesson times and differentiates the semesters, where each lesson time of each semester is named period (15 periods distributed in 5 week days; each day is divided in first and second semester). Finally, the set $K = \{1, 2, \cdots, 16\}$ is applied to organize the subjects in categories by syllabus affinities.

The highest weekly workload for classes' distribution, in each semester of each professor, is obtained after administrative charges, undergraduate advising charges (like traineeships, scientific initiation and final paper) and graduate charges (classes and advising) are considered. After that, it is checked if there is space to assign undergraduate classes according to the contract of each professor (20 or 40 hours per week), so that a group of professors whose contracts have already exceeded the workload is discriminated, for whom no more than 4 hours are assigned. The highest weekly workload for classes' distribution of other professors is calculated proportionally, so that the final spreadsheet of each professor is extrapolated equally. Mathematically, these pieces of information are organized in $c_{\max_{is}}$ which is the highest weekly workload of each professor, $i \in I$, in the first or in the second semester ($s = 1$ or $s = 2$). The professors also inform their available schedule, in which $d_{it} = 1$ if teacher $i \in I$ is available in period $t \in T$ and $d_{it} = 0$ otherwise.

Each professor has to fill a spreadsheet indicating their preferences and skills in the 16 categories of subjects. Preferences (p_{ij}, $i \in I$ and $j \in J$) and skills (h_{ij}, $j \in J$ and $j \in J$) vary among: none (weight 0), low (weight 1), medium (weight 2) and high (weight 3).

In addition to the above information, the schedule of subjects under DMAT/ UEL's responsibility is known, mathematically represented by $c_{jt} = 1$ if the subject $j \in J$ is located in period $t \in T$ and, $c_{jt} = 0$ otherwise. Furthermore, $g_{jk} = 1$ if subject $j \in J$ belongs to category $k \in K$ and $g_{jk} = 0$ otherwise. Finally, a_j represents the forecast of the number of students in subject $j \in J$.

3.2 Mathematical Model

The proposed model to perform the professor/subject assignment is a binary linear mathematical model, which intends to maximize preferences and skills of DMAT/UEL professors.

Since the final decision must answer which professor will perform which subject, a decision variable must represents the professor/subject assignment, that is, $x_{ij} = 1$ if professor $i \in I$ is assigned to subject $j \in J$ and $x_{ij} = 0$ otherwise. Another required decision variable consists of defining which professors will perform in which subject categories, that is, $y_{ik} = 1$ if professor $i \in I$ has received a subject from category $k \in K$, and $y_{ik} = 0$ otherwise.

Considering these conditions, the proposed mathematical model is described as follows.

Maximize

$$z = \sum_{i \in I} \sum_{j \in J} (\lambda h_{ij} + (1 - \lambda) p_{ij}) x_{ij} \ , \tag{1}$$

subject to:

$$(d_{it} - c_{jt})x_{ij} \geq 0,\ i \in I,\ j \in J, \text{ and } t \in T \tag{2}$$

$$\sum_{j \in J} a_j x_{ij} \leq a_{\max},\ i \in I \tag{3}$$

$$\sum_{i \in I} \sum_{t \in T} c_{jt} x_{ij} \leq c_{\max_{is}},\ i \in I \text{ and } s \in \{1, 2\} \tag{4}$$

$$\sum_{j \in J} p_{ij} x_{ij} \geq p_{\min},\ i \in I \tag{5}$$

$$\sum_{j \in J} h_{ij} x_{ij} \geq h_{\min},\ i \in I \tag{6}$$

$$\sum_{i \in I} x_{ij} = 1,\ j \in J \tag{7}$$

$$\sum_{j \in J} x_{ij} \geq 1,\ i \in I \tag{8}$$

$$\sum_{j \in J} c_{jt} x_{ij} \leq 1,\ i \in I \text{ and } t \in T \tag{9}$$

$$\sum_{j \in J} g_{jk} x_{ij} \leq \mu y_{ik},\ i \in I \text{ and } k \in K \tag{10}$$

$$\sum_{k \in K} y_{ik} \leq y_{\max},\ i \in I \tag{11}$$

$$x_{ij} \in \{0, 1\},\ i \in I,\ j \in J \tag{12}$$

$$y_{ik} \in \{0, 1\},\ i \in I,\ k \in K \tag{13}$$

The objective function (1) is composed of a convex combination between skills and preferences, with $\lambda \in [0, 1]$. It corresponds to maximize the summation of $i \in I$ (professor) and $j \in J$ (subject of the category $k \in K$), where p_{ij} assumes the value of the preference that professor i assign to subject j. The element h_{ij} assumes the value of the skill that professor i assign to subject j. The higher the λ value, function (1) maximizes the skills of the professor; the lower the λ value, function (1) maximizes the preferences of the professor.

Inequality (2) guarantees that each professor not be assigned to subjects whose classes are located in periods in which the professor is not available. Restriction (3) has been imposed in order to not burden teachers, in relation to correct home works and exams, limiting with $a_{\max}$ the total number of students to be attended per professor. Restriction (4) guarantees that professors have respected their limit of weekly workload. Restrictions (5) and (6) define a minimum level of preference and skill, respectively, for each professor. The p_{ij} element assumes the weight of the interest of the professor i in relation to subject j. A minimum level of the preference (p_{min}) for any professor is given by (5) and a minimum level of their skill (h_{min}) is given by (6).

Restriction (7) defines that each subject will be attended by only one teacher. Inequalities (8) and (9) guarantee that each professor will be assigned to at least one subject and each professor will not be assigned to more than one subject in the same period.

Inequalities (10) and (11) limit the number of categories that each professor will perform, where μ and $y_{\max}$ are pre-defined parameters. Finally, (12) and (13) define the domain where solutions are searched.

4 Simulations

The mathematical model presented in the Section 3 was solved by using *solver* Xpress-MP 7.1 (Xpress-IVE 2.21.02 - Xpress Mosel 3.2.0 - Xpress Optimizer 21.01.00), which has found a solution to the model in few seconds.

The data used to develop this paper correspond to the reality of the mathematics department in study and do not present great annual variability. For the tests, the values of the model parameters were fitted according to DMAT/UEL necessities. Parameters $p_{\min}$, $h_{\min}$ and μ were fitted using the value 4; on the other hand, the maximum number of students $a_{\max}$ assigned to each professor was set to 150 and the maximum number of categories assigned to each professor is $y_{\max} = 3$. The maximum number of subjects assigned to each professor is $d_{\max} = 4$.

These tests were performed to fit the parameter λ which belongs to the objective function, in order to evaluate the most relevant weight to be assigned to preference and skill.

In order to do this evaluation, results obtained from mathematical model of Section 3 were compared with the manually assignment which is performed by leaders of the DMAT/UEL. In this perform was used a preference index, a skill index and a medium index in each assignment. The preference individual index of professor $i \in I$ is defined by

$$I_p(i) = \frac{\sum_{j \in J} p_{ij} x_{ij}}{\sum_{j \in J} 3x_{ij}} \tag{14}$$

It should be noted that $I_p(i)$ is the ratio of the sum of the subjects weights assigned to professor and the maximum weight if subjects assigned to professor were those of his/her preference majority. This index is between 0 and 1. Unitary index indicates that classes assigned to professor are those of his/her preference. The index is null in an extreme case, in which only classes not chosen by the professor are assigned to him, however, these classes fit well in professor schedule. The average provides overall preference satisfaction:

$$I_p = \frac{1}{m} \sum_{i=1}^{m} I_p(i) \ .$$

The skill index is defined by

$$I_h(i) = \frac{\sum_{j \in J} h_{ij} x_{ij}}{\sum_{j \in J} 3x_{ij}} \tag{15}$$

with average

$$I_h = \frac{1}{m} \sum_{i=1}^{m} I_h(i) \ .$$

Fig. 1 and Fig. 2 illustrate, respectively, results about preferences and skills in 11 scenarios.

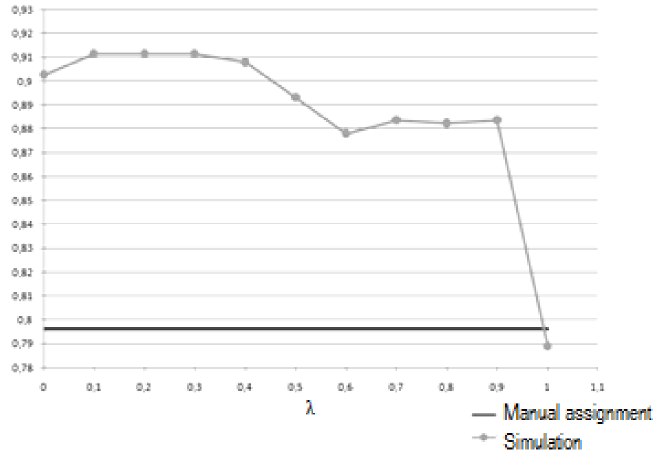

Fig. 1. Professors' average preference by simulating several values of λ.

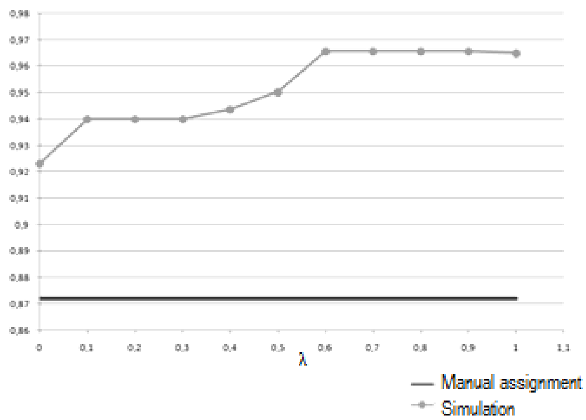

Fig. 2. Professors' average skill by simulating several values of λ.

As expected, average skill is a non-decreasing function of λ but surprisingly mean attended preferences is not non-increasing, showing a maximum for positive λ, $\lambda = 0.2$ in our simulations.

Various approaches are possible to qualify the best solution. First, the maximum of either average preferences or average skills. However, it seems natural to consider the best product of average preferences and average skills, which occur in the parameter range $[0.2, 0.3]$.

For $\lambda = 0,\ 0.1,\ 0.2,\ 0.3, 0.4$ and 0.9, the number of professors whose preferences were improved was 27. For $\lambda = 0.6, 0.7$ and 0.8, the number of professors whose preferences were improved was 26. For $\lambda = 0.5$, 24 professors had their preferences improved and, for $\lambda = 1$, 16 professors had their preferences improved. In the range in which there was the highest number of professors' preferences improvements, notice that $\lambda = 0.9$ provides the highest level of average skill (between 0.96 e 0.97). For $\lambda \in [0, 0.4]$ the level of average skill is between 0.94 and 0.95. With a view to increasing average preference, for $\lambda \in [0.1, 0.3]$, it is higher than 0.91, and for $\lambda = 0.9$, it is between 0.88 and 0.89.

From this information, the most relevant values to the DMAT/UEL are $\lambda \in [0.1, 0.3]$, where 27 professors had their preferences improved, the average index of professors' preferences I_p is higher than 0.93 and the average index of professors' skills I_h is 0.94.

5 Concluding Remarks

This work presented a case study of the timetable problem with data from the Mathematics Department at University of Londrina, aiming at the efficiency of the class-professor assignment and satisfaction of the majority of professors.

The mathematical model was performed with preferences and skills entering the objective function. The simulations conducted indicate that the model fulfill its purposes, and can be successfully adapted to other scenarios. A parameter conveniently switches preferences/skills objectives: for $\lambda = 0$ the model maximize preferences, whereas for $\lambda = 1$ the model maximize skills alone.

Scenarios as the one described in this paper are very difficult to be solved satisfactorily using only the human experience of a manager, because these situations involve people satisfaction and, on the other hand, the demand must be answered. In the university, there are general rules which must be obeyed by employees and which require an ethical posture from the manager. In problems like that, the help from Mathematics and Operational Research makes work easier and difficult tasks more manageable. In this way, the manager only has to fit a solution that, almost every time, is the most adequate one.

At the moment, the mathematics department, where the research described in this paper has been conducted, has enjoyed the simulations described in Section 4 as a support in decision making in relation to the workloads distributions. Since the results obtained and presented here are promising, the department has required the development of a prototype based on java to be available in an online management platform. For that purpose, we are developing a heuristic procedure that will be able to solve the mathematical model presented in this paper. For future papers, we propose an improvement in the objective function so that the excessive workload be equally distributed among professors.

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