1 Expanding Polynomial

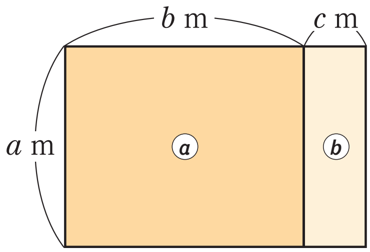
1 Multiplication and Division of Algebraic Expressions

Aim Let's consider multiplication and division of polynomial and monomial expressions.

Multiplication of Monomial and Polynomial Expressions

Q A rectangular plot of land has length a m and width b m. If we extend the width of this plot by c m, what will be its total area? Express your answer as an expression in the following two forms.

(1) (Length) × (Width)
(2) The sum of the areas ⓐ and ⓑ



For multiplication of monomial and polynomial expressions, we can use the distributive property, and remove the parentheses.

Mathematical Thinking 1

For multiplication of monomial and polynomial expressions, we can use the same method as for the multiplication of numbers and polynomial expressions.

Ex. 1

(1) $3x(x+5)$
$=3x\times x+3x\times 5$
$=3x^2+15x$

(2) $(5a-3)\times(-2a)$
$=5a\times(-2a)-3\times(-2a)$
$=-10a^2+6a$

Review

$a(b+c)=ab+ac$

$(b+c)a=ab+ac$

Junior High 1

(1) $3x\,(x+5)$

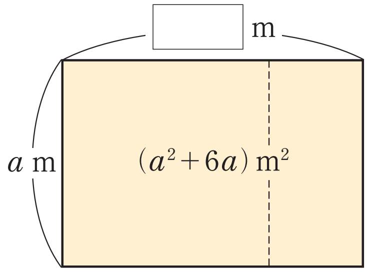
(2) $(5a-3)\times(\,-2a\,)$

Q 1 Calculate.

(1) $a(a+3)$
(2) $-4x(2x-5)$
(3) $(-3a+1)\times 6a$
(4) $(2x+4y)\times(-y)$
(5) $2a(a^2+2a-3)$
(6) $(6x-9)\times\dfrac{2}{3}x$

Division of Polynomial and Monomial Expressions

Q A plot of land has length a m and area (a^2+6a) m^2. What is its width?



Ex. 2

(1) $(a^2+6a)\div a$

$$=(a^2+6a)\times\frac{1}{a}$$

$$=a^2\times\frac{1}{a}+6a\times\frac{1}{a}$$

$$=a+6$$

$$\frac{\overset{a}{\cancel{a^2}}}{\underset{1}{\cancel{a}}}+\frac{\overset{1}{6\cancel{a}}}{\underset{1}{\cancel{a}}}=a+6$$

(2) $(xy-4y^2)\div\frac{1}{2}y$

$$=(xy-4y^2)\times\frac{2}{y}$$

$$=xy\times\frac{2}{y}-4y^2\times\frac{2}{y}$$

$$=2x-8y$$

$\frac{1}{2}y=\frac{y}{2}$,
therefore, the reciprocal of
$\frac{1}{2}y$ is $\frac{2}{y}$.

$$\frac{\overset{1}{x\cancel{y}\times 2}}{\underset{1}{\cancel{y}}}-\frac{\overset{y}{4\cancel{y^2}\times 2}}{\underset{1}{\cancel{y}}}=2x-8y$$

For division of polynomials by monomials, simply change them to multiplication.

Q 2 Calculate.

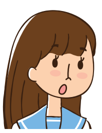
(1) $(10x^2+7x)\div x$

(2) $(8a^2b-2ab^2)\div 2ab$

(3) $(4x^2-6xy)\div\frac{2}{3}x$

(4) $(-2ab+a)\div\left(-\frac{a}{4}\right)$

Try it out
P.24
Enhancement 1-1



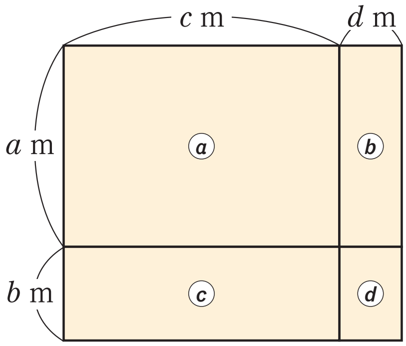
We can now do multiplication and division of polynomial and monomial expressions.

Can the multiplication of polynomials by polynomials also be done in the same way? P.16

2 Expanding Expression

Aim Let's consider multiplication of polynomials by polynomials.

Q A rectangle is shown on the right. Express its area using various expressions.



In **Q**, the total area can be expressed by (Length) × (Width) and ⓐ + ⓑ + ⓒ + ⓓ. From this, the following expression holds true.

$$(a+b)(c+d)=\underbrace{ac}_{ⓐ}+\underbrace{ad}_{ⓑ}+\underbrace{bc}_{ⓒ}+\underbrace{bd}_{ⓓ}$$

Ex. 1 In $(a+b)(c+d)$, if we consider $c+d$ to be one number and make $c+d=M$ using the distributive property, we can perform the following calculation.

$$\begin{aligned}&(a+b)(c+d)\\ =\;&(a+b)M\\ =\;&aM+bM\\ =\;&a(c+d)+b(c+d)\\ =\;&ac+ad+bc+bd\end{aligned}$$

Make $c+d=M$

Distributive property

Change M back to $c+d$

Distributive property

Q 1 Calculate $(a+b)(c+d)$ by letting $a+b=N$. Compare your results with Ex. 1

Generally, $(a+b)(c+d)$ can be calculated as shown on the right.

$$(a+b)(c+d)=\underbrace{ac}_{❶}+\underbrace{ad}_{❷}+\underbrace{bc}_{❸}+\underbrace{bd}_{❹}$$

Removing the parentheses from monomials, polynomials and the product of multiple polynomials, and expressing it as the sum of monomials, is called **expanding** the original expression.

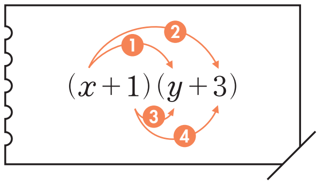
Ex. 2

(1) $(x+1)(y+3)$

$=xy+3x+y+3$

(2) $(a-3)(b+2)$

$=ab+2a-3b-6$



Q 2 Expand.

(1) $(a+3)(b+5)$ (2) $(x-2)(y+6)$

(3) $(a+b)(c-d)$ (4) $(x-a)(y-b)$

Ex. 3

(1) $(2x-5)(x+4)$

$=2x^2+8x-5x-20$

$=2x^2+3x-20$

(2) $(3x+2y)(2x-3y)$

$=6x^2-9xy+4xy-6y^2$

$=6x^2-5xy-6y^2$

When there are like terms in an expanded expression, combine them.

Q 3 Expand.

(1) $(x+1)(x+6)$ (2) $(x+2)(x-7)$

(3) $(x+6)(x-6)$ (4) $(3x-1)(x-5)$

(5) $(-a+4)(2a-5)$ (6) $(5x-y)(x+2y)$

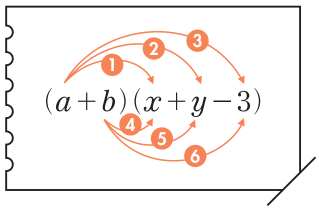
Ex. 4

$(a+b)(x+y-3)$

$=a(x+y-3)+b(x+y-3)$

$=ax+ay-3a+bx+by-3b$

Consider $x+y-3$ to be 1 number



Q 4 Expand.

(1) $(a-b)(x-y+2)$ (2) $(x+y+1)(x-y)$

Try it out

P.24 Enhancement 1-2

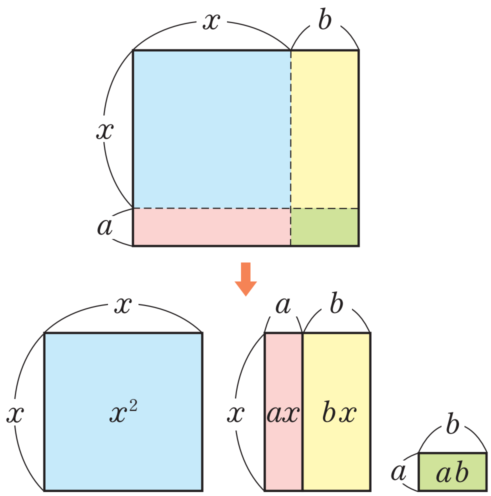
We can now expand various expressions.

Some of them are more frequently used in the expansion of polynomials. Let's summarize into a formula. P.18

3 Expansion Formula

Aim Let's summarize the most frequently used expansions of polynomials into a formula.

Formula for $(x+a)(x+b)$



Q Fill in the $\square$ with the appropriate expression.

$$(x+a)(x+b)$$
$$=x^2+\square x+\square x+ab$$
$$=x^2+(\square)x+ab$$

Formula 1 $(x+a)(x+b)=x^2+(a+b)x+ab$

Ex. 1

(1) $(x+2)(x+4)$

$=x^2+(2+4)x+2\times 4$

$=x^2+6x+8$

Sum

$(x+2)(x+4)=x^2+6x+8$

Product

(2) $(x+3)(x-5)$

$=x^2+\{3+(-5)\}x+3\times(-5)$

$=x^2-2x-15$

$(x+a)(x+b)$

$(x+3)(x-5)$

Q 1 Expand.

(1) $(x+2)(x+1)$

(2) $(y+5)(y+4)$

(3) $(a-5)(a+3)$

(4) $(a-7)(a-2)$

(5) $(x+8)(x-6)$

(6) $(x+3)(x-3)$

(7) $(y-1)(y-10)$

(8) $(x+3)^2$

(9) $\left(x+\frac{2}{3}\right)\left(x+\frac{1}{3}\right)$

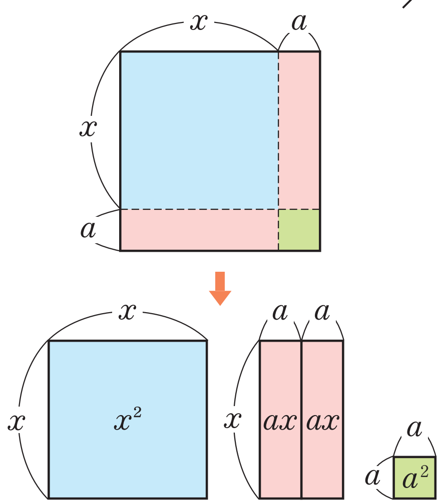
(10) $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{2}\right)$

Formula for the Square of a Polynomial

Q In the expression from **Q** on the previous page, what happens if we change b into a? Fill in the following $\square$ with the appropriate expression.

$$
\begin{aligned}
&(x+a)^2\\
&=(x+a)(x+a)\\
&=x^2+\square x+\square x+a^2\\
&=x^2+\boxed{}\,x+a^2
\end{aligned}
$$

$$(x+a)(x+b)$$
$$\downarrow$$
$$(x+a)(x+a)$$



$(x-a)^2$ can also be expanded like in **Q**.

$$
\begin{aligned}
&(x-a)^2\\
&=(x-a)(x-a)\\
&=x^2-ax-ax+a^2\\
&=x^2-2ax+a^2
\end{aligned}
$$

Formula ② $(x+a)^2=x^2+2ax+a^2$ (Square of a sum)

Formula ③ $(x-a)^2=x^2-2ax+a^2$ (Square of a difference)

Ex. 2

(1) $(x+3)^2$

$$
\begin{aligned}
&=x^2+2\times3\times x+3^2\\
&=x^2+6x+9
\end{aligned}
$$

(2) $(x-5)^2$

$$
\begin{aligned}
&=x^2-2\times5\times x+5^2\\
&=x^2-10x+25
\end{aligned}
$$

Double

$(x+3)^2=x^2+6x+9$

Square

Double

$(x-5)^2=x^2-10x+25$

Square

Q 2 Expand.

(1) $(x+1)^2$ (2) $(y+7)^2$ (3) $(x-2)^2$

(4) $(a-9)^2$ (5) $(a+b)^2$ (6) $\left(x-\dfrac{1}{2}\right)^2$

Formula for the Product of the Sum and the Difference

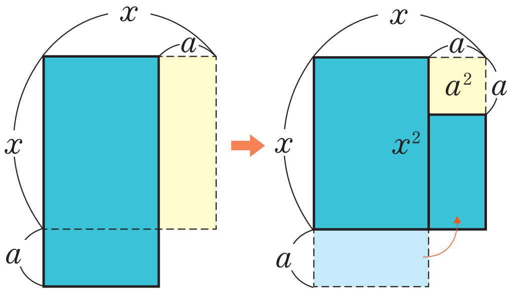
$(x+a)(x+b)$

$\downarrow$

$(x+a)(x-a)$

Q For the expression in **Q** on page 18, what happens if we change b into $-a$? Fill in the $\square$ with the appropriate expression.

$$\begin{aligned}&(x+a)(x-a)\\=\ &x^2-\square x+\square x-a^2\\=\ &x^2-a^2\end{aligned}$$



Formula 4 $(x+a)(x-a)=x^2-a^2$

Ex. 3

$$\begin{aligned}&(x+3)(x-3)\\=\ &x^2-3^2\\=\ &x^2-9\end{aligned}$$

Q 3 Expand.

(1) $(x+2)(x-2)$ (2) $(x-8)(x+8)$

(3) $(3+y)(3-y)$ (4) $(a-b)(a+b)$

(5) $(x-5)(5+x)$ (6) $\left(x+\dfrac{1}{3}\right)\left(x-\dfrac{1}{3}\right)$

Let me ask!

Can we divide a polynomial by a polynomial?

P.23

Try it out

P.24
Enhancement 1-3

We call formulas 1 ~ 4 **expansion formulas.**

IMPORTANT

Expansion Formulas

1. $(x+a)(x+b)=x^2+(a+b)x+ab$
2. $(x+a)^2=x^2+2ax+a^2$
3. $(x-a)^2=x^2-2ax+a^2$
4. $(x+a)(x-a)=x^2-a^2$

Various Calculations

Using the expansion formulas, try performing various calculations.

Q Expand the following expression.

$$(3x+1)(3x+7)$$

When the coefficient of x is not 1, such as in $(3x+1)(3x+7)$, if we consider $3x$ to be one number and make $3x=A$, we can use formula ① and calculate in the following way.

$$
\begin{aligned}
&(3x+1)(3x+7) && \text{Make } 3x=A\\
&=(A+1)(A+7) && \text{Expand}\\
&=A^2+8A+7 && \text{Change } A \text{ back to } 3x\\
&=(3x)^2+8\times 3x+7\\
&=9x^2+24x+7
\end{aligned}
$$

Ex. 4 Expand $(4x-3y)^2$.

Solution

$$
\begin{aligned}
&(4x-3y)^2\\
&=(4x)^2-2\times 4x\times 3y+(3y)^2\\
&=16x^2-24xy+9y^2
\end{aligned}
$$

Answer $16x^2-24xy+9y^2$

Q 4 Expand.

(1) $(3a+2)(3a+5)$ (2) $(5a-4)(5a+6)$

(3) $(2x+5)^2$ (4) $(4x-y)^2$

(5) $(3x-1)(3x+1)$ (6) $(6a+7b)(6a-7b)$

Q 5 Yugo performed the expansion of $(5x-3)^2$ as shown on the right. Is this expansion correct? If there are mistakes, correct them.

Is this correct?

$$
\begin{aligned}
&(5x-3)^2\\
&=(5x)^2-2\times 3\times x+3^2\\
&=25x^2-6x+9
\end{aligned}
$$

Ex. 5 Expand $(x+y+5)(x+y-2)$.

Method Make $x+y=M$, and expand using formula ①.

$$\begin{aligned}(x+y+5)(x+y-2) &= (x+y)^2+3(x+y)-10\\ (M+5)(M-2) &= M^2+3M-10\end{aligned}$$

Solution

Making $x+y=M$,

$$\begin{aligned}&(x+y+5)(x+y-2)\\ &=(M+5)(M-2)\\ &=M^2+3M-10\\ &=(x+y)^2+3(x+y)-10\\ &=x^2+2xy+y^2+3x+3y-10\end{aligned}$$

Change M back to $x+y$

Answer $x^2+2xy+y^2+3x+3y-10$

When expanding expressions, there are times when you can use the expansion formulas by grouping one part of the expression and substituting it with a letter, such as in Ex. 5.

Q 6 Expand.

(1) $(x+y+4)(x+y+1)$ (2) $(x-y-3)(x-y-6)$

(3) $(a-b+3)^2$ (4) $(a+b-7)(a+b+7)$

Ex. 6

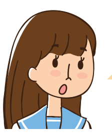
$$\begin{aligned}&(x-3)^2-(x-5)(x+4)\\ &=(x^2-6x+9)-(x^2-x-20)\\ &=x^2-6x+9-x^2+x+20\\ &=-5x+29\end{aligned}$$

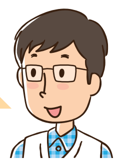
Q 7 Calculate.

(1) $x^2+(x+5)(x+1)$ (2) $(a+4)^2-(a-2)(a+2)$

(3) $(y+2)(y-7)-y(y-4)$ (4) $2(x-1)^2-(2x-1)^2$

Try it out
▶ P.24
Enhancement 1-4





Let's Check

1 Multiplication and Division of Algebraic Expressions
[P.14] Ex.1
[P.15] Ex.2

Calculate.

(1) $x(2x+5y)$ (2) $2x(3x-4y)$

(3) $(6a^2-7a)\div a$ (4) $(12a^2+9a)\div 3a$

2 Expanding Expression
[P.17] Ex.2
Ex.3

Expand.

(1) $(x+2)(y+5)$ (2) $(2x+1)(x-4)$

3 Expansion Formula
[P.18] Ex.1
[P.19] Ex.2
[P.20] Ex.3

Expand.

(1) $(a+5)(a+9)$ (2) $(x-7)(x+3)$ (3) $(y-1)(y-8)$

(4) $(a+8)^2$ (5) $(x-3)^2$ (6) $(y-4)(y+4)$

4 Various Calculations
[P.22] Ex.6

Calculate $(x+1)^2+(2+x)(2-x)$.

close up

Dividing Polynomials by Polynomials

Level UP!

We can consider the division of polynomials by polynomials by applying what we have learned about the division of integers and decimals.

For example, By making

$(x^2+3x-10)\div(x-2)$

into the form on the right, we can see that the quotient is $x+5$.

$$\begin{array}{r|l}
 & x \\
x-2 & x^2+3x-10 \\
 & x^2-2x \\
\hline
 & \quad 5x
\end{array}$$

① Write x

② $(x-2)\times x$

③ $(x^2+3x)-(x^2-2x)$

$$\begin{array}{r|l}
 & x \;+5 \\
x-2 & x^2+3x-10 \\
 & x^2-2x \\
\hline
 & \quad 5x-10 \\
 & \quad 5x-10 \\
\hline
 & \qquad\quad 0
\end{array}$$

⑤ Write 5

④ Bring down -10

⑥ $(x-2)\times 5$

⑦ $(5x-10)-(5x-10)$

Try and calculate $(3x^2+5x-12)\div(x+3)$.

Enhancement 1

→ **Expanding Polynomial**

Let's use what we have learned for home study and calculation practice.

1 Multiplication and Division of Algebraic Expressions

(1) $2x(x+4)$

(2) $-x(6-3x)$

(3) $(-5a+8)\times 2a$

(4) $(7x-2)\times(-4x)$

(5) $-3a(a-5b+1)$

(6) $(12a+8)\times\dfrac{3}{4}a$

(7) $(2x^2-9x)\div x$

(8) $(15a^2+3ab)\div 3a$

(9) $(4a^2b-ab^2)\div ab$

(10) $(8x^2+6xy)\div(-2x)$

(11) $(-3xy+2x)\div\left(-\dfrac{x}{3}\right)$

2 Expanding of Expression

(1) $(a+8)(b+2)$

(2) $(x-7)(y+6)$

(3) $(2a-1)(a-8)$

(4) $(4+2x)(3x+1)$

(5) $(2a-5b)(-a+6b)$

(6) $(7x+2y)(-7x+3y)$

(7) $(a+b)(x-y+5)$

(8) $(a-2b)(x+2y-3)$

(9) $(x+y-3)(x-y)$

(10) $(2a-b-4)(a+3b)$

3 Expansion Formula

(1) $(x+3)(x+7)$

(2) $(x-4)(x-5)$

(3) $(x+9)(x-10)$

(4) $(x-1)(x+6)$

(5) $(x+4)^2$

(6) $(x-10)^2$

(7) $(a-b)^2$

(8) $\left(x+\dfrac{1}{3}\right)^2$

(9) $(x+1)(x-1)$

(10) $(a-9)(a+9)$

(11) $(6+x)(6-x)$

(12) $\left(x+\dfrac{5}{4}\right)\left(x-\dfrac{5}{4}\right)$

4 Various Calculations

(1) $(2x-7)(2x+7)$

(2) $(3a+5)^2$

(3) $(4x-3y)^2$

(4) $(2a+6)(2a+3)$

(5) $(x-y+8)(x-y-8)$

(6) $(a+b-2)(a+b-5)$

(7) $(a+b-4)(a-b+4)$

(8) $(x+3)^2-x(x-4)$

(9) $b^2+(a+b)(a-b)$

(10) $(x+3)(x+4)-(x-2)(x+2)$

(11) $(2a+b)^2-(2a-b)^2$

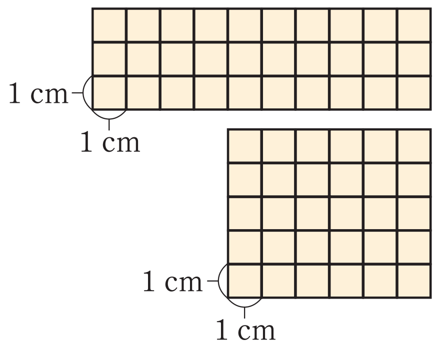
Answers on P.284, 285

2 Factoring

1 Prime Factorization

Aim Let's investigate whether we can express natural numbers as a product of several natural numbers.

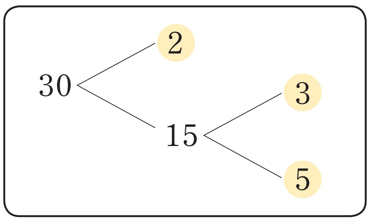
Q Make a rectangle by arranging 30 squares having a side of 1 cm. Think about the possible length and width.

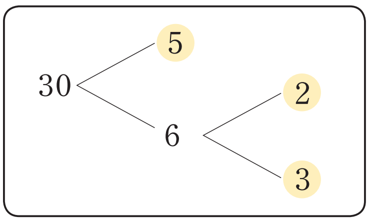


When we express a natural number as a product of several natural numbers, each of these natural numbers is called a **factor** of the original natural number.
For example, we can express $30=3\times10$, so 3 and 10 are factors of 30.
For natural numbers not including 1 and prime numbers, we can express them as a product of prime numbers, such as in $30=2\times3\times5$.
The factors that are prime numbers are called **prime factor** of the original natural number, and expressing a natural number as the product of its prime factors is called **prime factorization**.

> **Review**
> Natural numbers whose only divisors are 1 and itself are called prime numbers.
> Elementary 5

For prime factorization, the result will be the same regardless of which order you perform it.





Ex. 1 Find the prime factors of 150.

Method Sequentially divide using prime numbers, until the quotient becomes a prime number, as shown on the right.

$$\begin{array}{r|r} 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

Solution

$150 = 2 \times 3 \times 5 \times 5$

$= 2 \times 3 \times 5^2$ Answer $2 \times 3 \times 5^2$

Q 1 Find the prime factors of the following numbers.

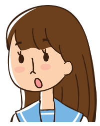
(1) 24 (2) 32 (3) 75 (4) 132

Q 2 Squaring a certain natural number gives 1764. Using prime factorization, find this natural number.

We can now do prime factorization of natural numbers.

How can we also express polynomials as a product of a number of expressions?

P.27



close up

How to Find the Divisors

We can find the divisors of large numbers using prime factorization. For example, the divisors of 135 can be found by prime factorization where $135 = 3^3 \times 5$. Using the following tree diagram, we can find all the divisors.

Divisors of 3^3	Divisors of 5		Divisors of 135
1	1	$1 \times 1 =$	1
	5	$1 \times 5 =$	5
3	1	$3 \times 1 =$	3
	5	$3 \times 5 =$	15
3^2	1	$3^2 \times 1 =$	9
	5	$3^2 \times 5 =$	45
3^3	1	$3^3 \times 1 =$	27
	5	$3^3 \times 5 =$	135

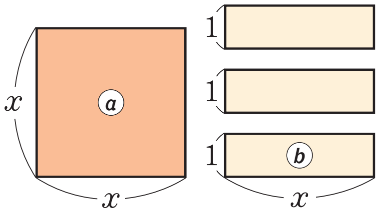
Using prime factorization, find all the divisors of 200.

2 Factoring

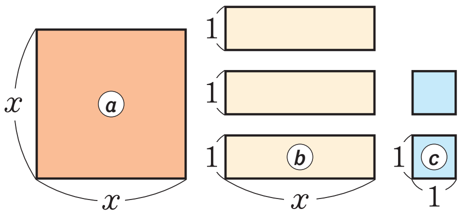
Aim Let's express the polynomials as products of several expressions.

Q Rearrange the square and the rectangular pieces of paper to make 1 rectangle. Cut out and use the figures from Appendix ②.

(1) Rearrange the following square and rectangular pieces of paper to make 1 rectangle.



(2) Rearrange the following squares and rectangular pieces of paper to make 1 rectangle.

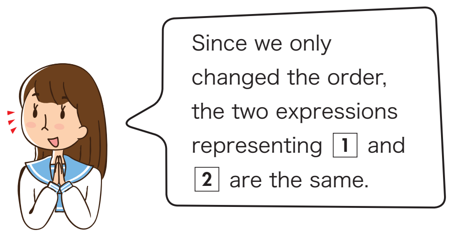


(3) Using 1 piece of ⓐ and several pieces of ⓑ and ⓒ, make 1 rectangle.

(4) For each of (1) ~ (3) above, represent $\boxed{1}$ and $\boxed{2}$, in an expression.

$\boxed{1}$ Sum of the area of the squares and rectangles before rearranging

$\boxed{2}$ Area of the rectangle formed after rearranging.



Within polynomials, there are those that can be expressed as a product of several polynomials. For example, for (1) and (2) in Q on the previous page, the following holds true.

$$x^2+3x=x(x+3) \qquad ①$$

$$x^2+3x+2=(x+1)(x+2) \qquad ②$$

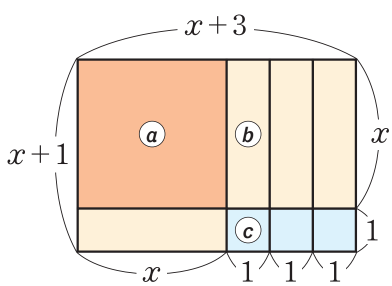
When a polynomial expressed as a sum of several monomials is expressed as a product of polynomials, each of the expressions is a **factor** of the original polynomial.
For example, for ①, x and $x+3$ are the factors of the polynomial x^2+3x, and for ②, $x+1$ and $x+2$ are the factors of x^2+3x+2.

Ex. 1

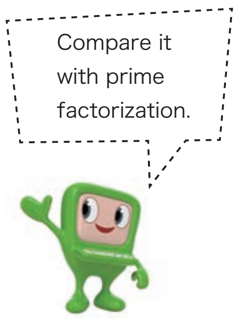
In (3) from Q on the previous page, if we use 1 piece of ⓐ, 4 pieces of ⓑ, and 3 pieces of ⓒ to make a rectangle like the one on the right, the following expression holds true.

$$x^2+4x+3=(x+1)(x+3)$$

In this case, $x+1$ and $x+3$ are the factors of x^2+4x+3.



Expressing a polynomial as a product of its factors is called **factoring** the polynomial.



$$x^2+3x+2 \underset{\text{Expanding}}{\overset{\text{Factoring}}{\rightleftarrows}} (x+1)(x+2)$$

Form as a sum of monomials | Form as a product of factors

Q 1 Which of the following expressions has been factored?

ⓐ $x^2-5x=x(x-5)$

ⓑ $x^2+7x+12=x(x+7)+12$

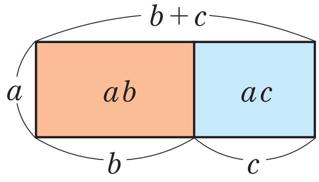
ⓒ $x^2+6x+8=(x+3)^2-1$

ⓓ $x^2-9=(x+3)(x-3)$

Common Factors

Consider the factored expressions from ① on the previous page. When there is a common factor among the terms in a polynomial, we can use the distributive property to take out the common factor and place it outside the parentheses before factoring the polynomial.

$$ab + ac = a(b+c)$$



Ex. 2

(1) $mx-my$
$=m(x-y)$

(2) $ax^2+2ax+7a$
$=a(x^2+2x+7)$

Q 2

Factor.

(1) $ax+bx$ (2) $ax-a$ (3) $px^2-5px+3p$

Ex. 3

Factor the polynomial $2a^2+4ab$.

Method

$2a^2=2a\times a$, $4ab=2a\times 2b$
Therefore, $2a$ is a common factor of both terms.

$$2a^2+4ab$$
$$=2a\times a+2a\times 2b$$

Solution

$2a^2+4ab$
$=2a\times a+2a\times 2b$
$=2a(a+2b)$ Answer $2a(a+2b)$

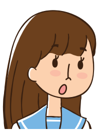
Note When factoring $2a^2+4ab$, instead of leaving them as simply $2(a^2+2ab)$ or $a(2a+4b)$, take out the common factor and place it outside the parentheses.

Q 3

Factor.

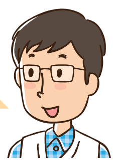
(1) $4ax+8ay$ (2) $3x^2+7x$
(3) x^2-x (4) x^2y+xy^2
(5) $a^2+6ab-8a$ (6) $9x^2-3xy+6x$

Try it out
P.35 Enhancement 2-1



We can now factor polynomials with common factors.

Think about factoring expressions like ② from the previous page. P.30



3 Factoring using Formulas

Aim Let's consider factoring polynomials using the expansion formulas.

①′ $x^2+(a+b)x+ab=(x+a)(x+b)$

Ex. 1 Factor x^2+6x+8.

$$x^2+(a+b)x+ab$$
$$x^2+\quad 6x \quad +\ 8$$

Method Find 2 numbers whose product is 8 and whose sum is 6.

① There are 4 pairs of integers whose product is 8, as shown in the table on the right.

② Among these pairs, 2 and 4 have a sum of 6.

Product is 8	Sum is 6
1 and 8	×
−1 and −8	×
2 and 4	○
−2 and −4	×

Solution

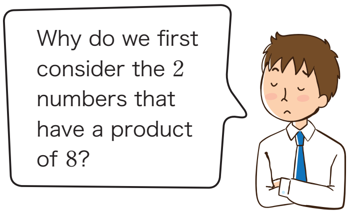
x^2+6x+8

$=x^2+(2+4)x+2\times4$

$=(x+2)(x+4)$

Answer $(x+2)(x+4)$

Note The answer can be written as either $(x+2)(x+4)$ or $(x+4)(x+2)$.



Q 1 Factor.

(1) x^2+5x+6 (2) x^2+9x+8

(3) $x^2-7x+10$ (4) x^2-5x+4

Ex. 2 Factoring x^2+3x-4, among the pairs of numbers that give a product of -4, find the two numbers whose sum is 3.

x^2+3x-4

$=(x-1)(x+4)$

Product is −4	Sum is 3
1 and −4	×
−1 and 4	○
−2 and 2	×

Q 2 Factor.

(1) x^2+x-12 (2) x^2+2x-3

(3) $x^2-2x-15$ (4) x^2-4x-5

②′ $x^2+2ax+a^2=(x+a)^2$, ③′ $x^2-2ax+a^2=(x-a)^2$

Ex. 3 Factor x^2+6x+9.

Method Since $9=3^2$, and $6=2\times3$, we will factor using the formula for the square of a polynomial.

$$x^2+\ \ 2\,a\,x\ \ +a^2=(x+a)^2$$
$$x^2+2\times3\times x+3^2=(x+3)^2$$

Solution

x^2+6x+9

$=x^2+2\times3\times x+3^2$

$=(x+3)^2$ Answer $(x+3)^2$

Q 3 Factor.

(1) x^2+2x+1 (2) x^2-2x+1

(3) x^2+4x+4 (4) $x^2-8x+16$

(5) $a^2+12a+36$ (6) $y^2-14y+49$

④′ $x^2-a^2=(x+a)(x-a)$

Ex. 4

x^2-16

$=x^2-4^2$

$=(x+4)(x-4)$

$$x^2-a^2=(x+a)(x-a)$$
$$x^2-4^2=(x+4)(x-4)$$

Q 4 Factor.

(1) x^2-9 (2) x^2-36

(3) $1-x^2$ (4) a^2-b^2

Q 5 Using formulas ①′ ~ ④′ that you have learned so far, factor the following.

(1) $x^2+8x+12$ (2) x^2-4x+4

(3) x^2-x-20 (4) x^2-100

(5) $x^2+18x+81$ (6) $x^2+3x-28$

Try it out
▶ P.35 Enhancement ②-2

Various Factorizations

Ex. 5

(1) $4x^2-12x+9$

$=(2x)^2-2\times 2x\times 3+3^2$

$=(2x-3)^2$

$$(2x)^2-2\times 2x\times 3+3^2=(2x-3)^2$$
$$A^2-2AB+B^2=(A-B)^2$$

(2) $9x^2-4y^2$

$=(3x)^2-(2y)^2$

$=(3x+2y)(3x-2y)$

$$(3x)^2-(2y)^2=(3x+2y)(3x-2y)$$
$$A^2-B^2=(A+B)(A-B)$$

Q 6

Factor.

(1) $4x^2+4x+1$

(2) $9x^2-12x+4$

(3) $x^2+2xy+y^2$

(4) $x^2-6xy+9y^2$

(5) $25b^2-9a^2$

(6) $x^2-\dfrac{y^2}{4}$

Ex. 6

Factor $ax^2-2ax-8a$.

Method First, take the common factors out of the parentheses, then consider whether it can be factored further.

Solution

$ax^2-2ax-8a$

$=a(x^2-2x-8)$ — Take the common factor a out of the parentheses.

$=a(x+2)(x-4)$ — Factor the expression within the parentheses.

Write down your explanation of the calculation.

Answer $a(x+2)(x-4)$

Q 7

Factor.

(1) $ax^2-ax-2a$

(2) xy^2-x

(3) $2x^2+16x+32$

(4) $-3x^2+12xy-12y^2$

Ex. 7 Factor $(x+5)^2-(x+5)$.

Method Replace $x+5$ with a letter.

Solution

Making $x+5=M$,

$$\begin{aligned}&(x+5)^2-(x+5)\\ &=M^2-M\\ &=M(M-1)\\ &=(x+5)(x+5-1)\\ &=(x+5)(x+4)\end{aligned}$$

Take the common factor M out of the parentheses

Change M back to $x+5$

Answer $(x+5)(x+4)$

When we factor polynomial expressions such as in Ex. 7, there are times when we can use the distributive property or a formula, by grouping one part of the expression and replacing it with a letter.

Q 8 Factor.

(1) $(x-1)^2-(x-1)$

(2) $(a+b)x+(a+b)y$

(3) $(x+7)^2+6(x+7)-16$

(4) $(x+y)^2-81$

Ex. 8 Factor $xy+x+y+1$.

Method Consider the terms that contain x and the terms that do not contain x separately.

Solution

$$\begin{aligned}&xy+x+y+1\\ &=(xy+x)+(y+1)\\ &=x(y+1)+(y+1)\\ &=(y+1)(x+1)\end{aligned}$$

Take the common factor $y+1$ out of the parentheses

Answer $(y+1)(x+1)$

Check whether you get the same results if you separate the terms that contain y and the terms that do not contain y.

Q 9 Factor.

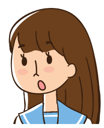
(1) $xy-x+y-1$

(2) $ax+3x-a-3$

Try it out
P.35
Enhancement 2-3

By using the expansion formulas, we can now factor various polynomial expressions.

Where can we use the things we have learned so far such as expanding expressions and factorization? P.36



Let's Check

1 Prime Factorization
[P.26] Ex.1

Factor 90.

2 Common Factors
[P.29] Ex.2 Ex.3

Factor.

(1) $7ax+2ay-9a$　　(2) $12x^2-8xy$

3 Factoring Using Formulas
[P.30] Ex.1 Ex.2
[P.31] Ex.3 Ex.4

Factor.

(1) x^2+7x+6　　(2) x^2-x-12　　(3) $x^2+10x+25$

(4) $x^2-16x+64$　　(5) x^2-81　　(6) $9-a^2$

4 Various Factorizations
[P.32] Ex.5 Ex.6
[P.33] Ex.7

Factor.

(1) $x^2-4xy+4y^2$　　(2) $36-9a^2$

(3) $ax^2+4ax-12a$　　(4) $(a+b)x-(a+b)y$

close up

Talking About Prime Factors

Find the prime factors up to 100 using the method below.

Write out the natural numbers like in the figure on the right, and cross out 1 first. Next, skip 2 and cross out all the multiples of 2. Then, skip 3 and cross out all the multiples of 3. Do the same for the remaining numbers, skipping the first number and crossing out all of its multiples. In the end, we will be left with 2, 3, 5, 7, 11, ⋯, a total of 25 prime numbers.

~~1~~	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100		

This method originated from ancient Greece, by a man named Eratosthenes (c. 275 ~ c. 194 BC). It is known as the Sieve of Eratosthenes.

Enhancement 2

→ **Factoring**

Let's use what we have learned for home study and calculation practice.

Factor.

1 Common Factors

(1) $xy+4x$

(2) $5ax-8ay+2a$

(3) x^2+7x

(4) $2x^2y-3xy^2$

(5) $6a^2+9ab$

(6) $10x^2-25xy+5x$

2 Factoring Using Formulas

(1) x^2+6x+5

(2) $x^2+10x+21$

(3) x^2-7x+6

(4) $x^2-12x+27$

(5) x^2+2x-8

(6) $x^2-3x-10$

(7) x^2-x-2

(8) $x^2+4x-45$

(9) $x^2+14x+49$

(10) $x^2+16x+64$

(11) $x^2-10x+25$

(12) $x^2-20x+100$

(13) x^2-1

(14) x^2-64

3 Various Factorizations

(1) $4x^2+12x+9$

(2) $9x^2-6x+1$

(3) $x^2-2xy+y^2$

(4) $x^2+8xy+16y^2$

(5) $100x^2-49$

(6) $16-25x^2$

(7) $4x^2-49y^2$

(8) $x^2-\frac{y^2}{9}$

(9) ax^2-ay^2

(10) $ax^2+2ax+a$

(11) $3x^2-18xy+27y^2$

(12) $2x^2y+4xy-30y$

(13) $x(x+3)-18$

(14) $(x-5)(x-2)+2$

(15) $(x+5)(x+1)+4$

(16) $(x+1)(x-4)-14$

(17) $(x+3)^2-2(x+3)$

(18) $(a-b)x+(a-b)y$

(19) $(x+2)^2+(x+2)-12$

(20) $(x-5)^2-25$

(21) $xy-5x+y-5$

(22) $2xy-3x+2y-3$

Answers on P.285