1 Simplifying Algebraic Expression

1 Structure of Algebraic Expressions

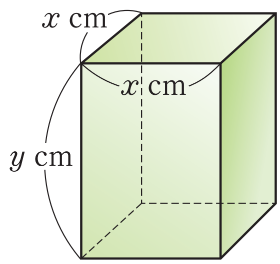
Aim Let's categorize and organize algebraic expressions.

Monomial Expressions and Polynomial Expressions

Q Communicate

The following expressions ⓐ ~ ⓕ represent various quantities from the square prism on the right.

ⓐ $4x$　ⓑ x^2　ⓒ $2x+2y$
ⓓ xy　ⓔ $2x^2+4xy$　ⓕ x^2y



(1) Consider what quantities these expressions represent. Consider what their units are.

(2) Discuss how we can categorize these expressions according to their characteristics.

Expressions that take the form of multiples of numbers or letters such as $4x$ and xy from **Q**, are called **monomial expressions**. Single letters or numbers such as y and -6 are also called monomial expressions.

Monomial Expressions $\begin{cases} 4x, \quad xy \\ y, \quad -6 \end{cases}$

Polynomial Expressions $\begin{cases} 10x+20 \\ 2x+2y \end{cases}$

Expressions that take the form of sums of monomials such as $10x+20$ and $2x+2y$ are called **polynomial expressions**. Each monomial expression in the polynomial expression is called a term of the polynomial expression.

Ex. 1 In the polynomial expression x^2-4x+3, x^2, $-4x$, and 3 are the terms of this expression.

x^2-4x+3

$=x^2+(-4x)+3$

Terms

Constant term

The numerical-only term of a polynomial is called the **constant term**.

Q 1 Divide the expressions in ⓑ, ⓔ, and ⓕ from **Q** into monomial expressions and polynomial expressions.

Q 2 State the terms of the following polynomials.

(1) $5a+1$　(2) $7x-8y$　(3) $4x^2+7x-9$

Degrees of Algebraic Expressions

Q Express the following monomial expressions using the multiplication sign $\times$.

(1) $2x$ (2) $-3x^2$ (3) $5x^2y$

The number of variables being multiplied in a monomial expression is called the **degree** of the monomial expression.

Ex. 2 The degrees of the monomial expressions (1)~(3) from **Q** are as follows.

(1) $2x$ …… The degree is 1

(2) $-3x^2$ …… The degree is 2

(3) $5x^2y$ …… The degree is 3

(1) $2x = 2 \times x$

(2) $-3x^2 = -3 \times x \times x$

(3) $5x^2y = 5 \times x \times x \times y$

Q 3 State the degrees of the following monomial expressions.

(1) $-6a$ (2) x^2 (3) $\frac{1}{2}ab$ (4) $-xy^2$

In polynomial expressions, the degree is the largest degree of its terms.

Note We can express the comparison of sizes of degrees using “larger” and “smaller”.

Ex. 3 In the polynomial expression $x^2 - 4x + 3$, the term with the largest degree is x^2 and its degree is 2, therefore the degree of $x^2 - 4x + 3$ is 2.

$x^2 \quad -4x \quad +3$

Degree 2 1

Constant term

An expression with a degree of 1 is called a linear expression, an expression with a degree of 2 is a quadratic expression, and so on.

Linear Expressions $\begin{cases} 2x, \ 5a+1, \\ x+8y-6 \end{cases}$

Quadratic Expressions $\begin{cases} -x^2, \ 7ab, \\ x^2-4x+3 \end{cases}$

Q 4 What are the degrees of ⓐ ~ ⓕ respectively from **Q** on the previous page?

So, for algebraic expressions, there are monomial expressions and polynomial expressions.

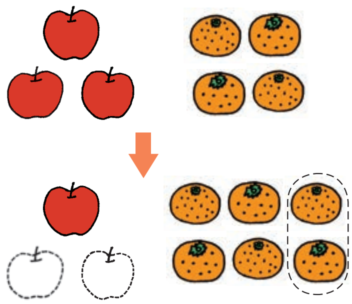
I wonder if we can calculate polynomial expressions with 2 variables in the same way as in Junior High 1? P.16

2 Simplifying Polynomial Expressions

• Aim • Let's consider how to calculate polynomial expressions with 2 letters.

Like Terms

Q I wanted to buy 3 apples at a yen each and 4 mandarins at b yen each. However, I did not have enough money so I reduced the number of apples by 2 and increased the number of mandarins by 2. Express the total cost using an expression.



The terms that have the same variable in an expression, such as $3a$ and $-2a$, or $4b$ and $2b$ in the polynomial expression $3a+4b-2a+2b$, are called **like terms**.

$$\underbrace{3a+4b-2a}_{\text{Like terms: } 3a,\ -2a}+2b \qquad 3a+\underbrace{4b-2a+2b}_{\text{Like terms: } 4b,\ 2b}$$

Q 1 State the like terms in the following polynomial expressions.

(1) $3x-4y-7x+2y$ (2) $a-6b-9b+3a$

Like terms can be combined into one term using the distributive property.

$$ma+na=(m+n)a$$

Ex. 1

(1)
$$\begin{aligned} &2x+8y-6x+y\\ &=2x-6x+8y+y\\ &=(2-6)x+(8+1)y\\ &=-4x+9y \end{aligned}$$

Change the order of the terms

Combine the like terms

(2)
$$\begin{aligned} &4a^2-7a+6a+3a^2\\ &=4a^2+3a^2-7a+6a\\ &=(4+3)a^2+(-7+6)a\\ &=7a^2-a \end{aligned}$$

Note The degrees of a^2 and a are different so they are not like terms.

Q 2 Combine the like terms.

(1) $5x+2y-3x+y$　　(2) $-7a+2b+6b-2a$

(3) $a-4b+7-3a+8b$　　(4) $4x^2+3x^2$

(5) $x^2+9x-8x^2-x$　　(6) $-3x^2-7x+3x^2+2x$

(7) $2x^2-6x-2-3x$　　(8) $x^2-8x+4-3x^2+8x$

Addition of Polynomial Expressions

Q Back in Junior High 1, how did you calculate linear expressions such as $(2x+4)+(x-2)$?

Ex. 2 Find the sum when $-3x+5y$ is added to $x-2y$.

$$\begin{array}{rr} & x-2y \\ +) & -3x+5y \\ \hline & -2x+3y \end{array}$$

Solution

$$\begin{aligned} &(x-2y)+(-3x+5y) \\ &= x-2y-3x+5y \\ &= x-3x-2y+5y \\ &= -2x+3y \end{aligned}$$

Answer $-2x+3y$

When calculating vertically, line up like terms.

When adding polynomial expressions, the sum can be simplified by adding the terms of the expressions and combining like terms.

Mathematical Thinking 1

You can consider calculation of polynomial expressions in the same way as calculation of algebraic expressions from Junior High 1.

Q 3 Find the sum when the expression on the right is added to the expression on the left in the following two expressions.

(1) $6a+4b,\ 3a+b$　　(2) $2x^2+6x,\ x^2-9x$

Q 4 Simplify.

(1) $(a+7b)+(4a-3b)$　　(2) $(-6x^2+5x-7)+(3x^2-5x)$

(3)
$$\begin{array}{rr} & 4x-\ \ y \\ +) & 2x+3y \\ \hline \end{array}$$

(4)
$$\begin{array}{rr} & 3x-\ \ y-5 \\ +) & -2x-4y+3 \\ \hline \end{array}$$

Subtraction of Polynomial Expressions

Q Fill in the $\square$ on the right with the appropriate sign. Find the answer to this calculation.

$$
\begin{aligned}
&(3x+1)-(2x-5)\\
&=(3x+1)+(\square\, 2x\, \square\, 5)\\
&=3x+1\, \square\, 2x\, \square\, 5
\end{aligned}
$$

Ex. 3 Find the difference when $3x-7y$ is subtracted from $5x-4y$.

Solution

$$
\begin{aligned}
&(5x-4y)-(3x-7y)\\
&=(5x-4y)+(-3x+7y)\\
&=5x-4y-3x+7y\\
&=2x+3y
\end{aligned}
$$

When subtracting expressions, make sure you use parentheses.

Answer $2x+3y$

$$
\begin{array}{rr}
 & 5x-4y\\
-) & 3x-7y\\
\hline
\end{array}
$$

$$\downarrow$$

$$
\begin{array}{rr}
 & 5x-4y\\
+) & -3x+7y\\
\hline
 & 2x+3y
\end{array}
$$

In subtraction of polynomial expressions, simply change the sign of the terms in the subtrahend and add them.

Q 5 In the following two expressions, find the difference when the expression on the right is subtracted from the expression on the left.

(1) $6a+4b,\ 3a+b$ (2) $2x^2+6x,\ x^2-9x$

Q 6 Simplify.

(1) $(4a-2b)-(a+5b)$ (2) $(x^2+3x+7)-(-6x^2-2x+5)$

(3)
$$
\begin{array}{rr}
 & 8x+7y\\
-) & x-2y\\
\hline
\end{array}
$$

(4)
$$
\begin{array}{rrrr}
 & x & +4y & -1\\
-) & 2x & & +6\\
\hline
\end{array}
$$

Try it out

P.25 Enhancement 1-1

Q 7 Communicate

Taichi looked at the notebook of his younger sister who is in Junior High 1. Show where the mistake is and explain why.

Is this correct?

$$
\begin{aligned}
&(4x+1)-(x-5)\\
&=4x+1-x+5\\
&=3x+6\\
&=9x
\end{aligned}
$$

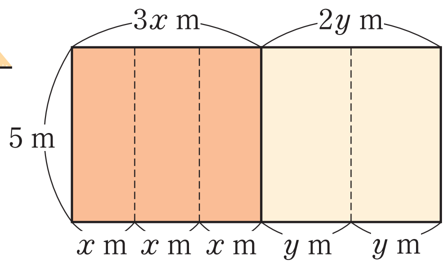
We were able to do addition and subtraction of polynomial expressions with 2 variables the same way as in Junior High 1.

Can we also do calculations such as $5(3x+2y)$ in the same way as in Junior High 1? P.19

Aim Let's consider multiplication and division of polynomial expressions and numbers.

Multiplication of Polynomial Expressions and Numbers

Q There is a plot of land in the shape of a rectangle as shown on the right. Express the total area of this plot of land using an expression.



Ex. 4

$$5(3x+2y)$$
$$=5\times 3x+5\times 2y$$
$$=15x+10y$$

Review

Distributive property

$$a(b+c)=ab+ac$$
$$(b+c)a=ab+ac$$

▶ Junior High 1

In multiplication of polynomial expressions and numbers, simply use the distributive property and remove the parentheses.

Q 8 Simplify.

(1) $3(x+5y)$ (2) $-4(-2a+b)$ (3) $(7a-4b)\times 5$

(4) $6(5x-2y+1)$ (5) $(3a+4b-5)\times(-2)$ (6) $\frac{1}{4}(-8x-2y)$

Division of Polynomial Expressions and Numbers

Ex. 5

$$(9x-15y)\div 3$$
$$=(9x-15y)\times\frac{1}{3}$$
$$=9x\times\frac{1}{3}-15y\times\frac{1}{3}$$
$$=3x-5y$$

Multiply by the reciprocal of the divisor

$$\frac{\overset{3}{\cancel{9}}x}{\underset{1}{\cancel{3}}}-\frac{\overset{5}{\cancel{15}}y}{\underset{1}{\cancel{3}}}=3x-5y$$

In division of polynomial expressions and numbers, simply change the form to multiplication.

Q 9 Simplify.

(1) $(10x-25y)\div 5$ (2) $(-12a+6b)\div(-3)$

Try it out

▶ P.25 Enhancement 1-2

Various Calculations

Ex. 6

$$4(3x+2y)-3(5x-y)$$
$$=12x+8y-15x+3y$$
$$=-3x+11y$$

When removing parentheses, be careful with the signs.

Q 10 Calculate.

(1) $2(a+2b)+3(2a-b)$

(2) $-3(4x-5y)+6(2x-3y)$

(3) $3(a-2b)-2(a+5b)$

(4) $7(x-2y+1)-4(-3y+2)$

Ex. 7

Method ①

$$\frac{x+2y}{2}-\frac{x-y}{3}$$

↓ Express them with a common denominator

$$=\frac{3(x+2y)}{6}-\frac{2(x-y)}{6}$$

↓ Combine into a single fraction

$$=\frac{3(x+2y)-2(x-y)}{6}$$

↓ Remove the parentheses in the numerator

$$=\frac{3x+6y-2x+2y}{6}$$

↓ Combine like terms

$$=\frac{x+8y}{6}$$

Method ②

$$\frac{x+2y}{2}-\frac{x-y}{3}$$

↓ Change into the form numerator × (polynomial expression)

$$=\frac{1}{2}(x+2y)-\frac{1}{3}(x-y)$$

↓ Remove the parentheses

$$=\frac{1}{2}x+y-\frac{1}{3}x+\frac{1}{3}y$$

↓ Rearrange the terms, and express them with a common denominator

$$=\frac{3}{6}x-\frac{2}{6}x+\frac{3}{3}y+\frac{1}{3}y$$

↓ Combine like terms

$$=\frac{1}{6}x+\frac{4}{3}y$$

Q 11 Calculate.

(1) $\dfrac{x+3y}{4}+\dfrac{3x-y}{6}$

(2) $\dfrac{x-y}{4}-\dfrac{2x+y}{8}$

(3) $\dfrac{1}{9}(5x+3y)-\dfrac{1}{3}(x-y)$

(4) $x+y-\dfrac{4x-2y}{5}$

Try it out

▶ P.25 Enhancement 1-3

In multiplication and division of polynomial expressions and numbers, we were able to use the distributive property as in Junior High 1.

Let's consider multiplication and division of monomial expressions.

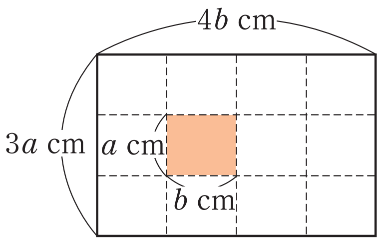
▶ P.21

3 Multiplication and Division of Monomial Expressions

Aim Let's consider multiplication and division for monomial expressions that contain variables.

Multiplication of Monomial Expressions that Contain Variables

Q Sheets of coloured paper of length a cm and width b cm are tiled on a rectangular mat of length $3a$ cm and width $4b$ cm. How many sheets of coloured paper are needed? What is the total area of the mat?



Ex. 1

$$\begin{aligned}
&3a\times 4b\\
&=(3\times a)\times(4\times b)\\
&=3\times 4\times a\times b\\
&=12ab
\end{aligned}$$

Product of the coefficients

$$3a\times 4b=12ab$$

Product of the variables

In multiplication of monomial expressions that contain variables, find the product of the coefficients and the variables, respectively, and simply combined them.

Q 1 Simplify.

(1) $5a\times 2b$ (2) $(-6x)\times 3y$ (3) $(-x)\times(-7y)$

(4) $0.4x\times(-5y)$ (5) $8a\times\frac{1}{4}b$ (6) $\left(-\frac{2}{3}x\right)\times(-9y)$

Ex. 2

(1) $3a^2\times 2a$

$=(3\times a\times a)\times(2\times a)$

$=3\times 2\times a\times a\times a$

$=6a^3$

(2) $(-5x)^2$

$=(-5x)\times(-5x)$

$=(-5)\times(-5)\times x\times x$

$=25x^2$

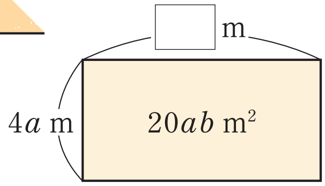
Q 2 Simplify.

(1) $a^3\times a^2$ (2) $2a^2\times 4a$ (3) $(3x)^2$

(4) $(-4a)^2$ (5) $(-6xy)\times 2y$ (6) $8x\times(-x)^2$

Division of Monomial Expressions that Contain Variables

Q A rectangular plot of land has length $4a$ m and area $20ab$ m^2. What is its width?



Ex. 3

(1) $20ab \div 4a$

$$= \frac{20ab}{4a}$$

$$= \frac{\overset{5}{\cancel{20}} \times \overset{1}{\cancel{a}} \times b}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{a}}}$$

$$= 5b$$

(2) $(-4x^2) \div \frac{1}{2}x$

$$= (-4x^2) \div \frac{x}{2}$$

$$= (-4x^2) \times \frac{2}{x}$$

$$= -\frac{4 \times \overset{1}{\cancel{x}} \times x \times 2}{\underset{1}{\cancel{x}}}$$

$$= -8x$$

The same variables can be reduced.

Q 3 Simplify.

(1) $12xy \div 6y$ (2) $(-9ab) \div 3b$ (3) $a^3 \div a^2$

(4) $10x^2y \div (-2xy)$ (5) $9x^2 \div \frac{3}{5}x$ (6) $4ab \div \left(-\frac{2}{3}b\right)$

Calculation of a Combination of Multiplication and Division

Ex. 4

$$4y^2 \div 6xy \times 12x$$

$$= 4y^2 \times \frac{1}{6xy} \times 12x$$

$$= \frac{4y^2 \times 12x}{6xy}$$

$$= 8y$$

$$\frac{4 \times \overset{1}{\cancel{y}} \times y \times \overset{2}{\cancel{12}} \times \overset{1}{\cancel{x}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{x}} \times \underset{1}{\cancel{y}}} = 8y$$

Q 4 Simplify.

(1) $3x^2 \times 4y \div 2xy$ (2) $x^3 \div 2x^2 \times 8x$

(3) $12a^2b \times (-3ab) \div 9ab^2$ (4) $27a^2 \div (-3a)^2$

Try it out
P.25 Enhancement Ⅰ-4

When finding the value of the expression, can we use the calculation of expressions we learned in Junior High 1?
P.23

In what situations can we use the algebraic expressions we have learned so far?
P.26, 31

4 Value of Expressions

Aim Let's consider how to find the value of expressions easily.

Value of Expressions

Q Communicate

For math problems such as the one below, Takumi and Yui found their answers as shown below.

When $x=-5$ and $y=4$, find the value of $7x-(6x-2y)$.

Takumi's Method

$$\begin{aligned}&7x-(6x-2y)\\&=7\times(-5)-\{6\times(-5)-2\times4\}\\&=-35-(-30-8)\\&=-35-(-38)\\&=-35+38\\&=3\end{aligned}$$

Yui's Method

$$\begin{aligned}&7x-(6x-2y)\\&=7x-6x+2y\\&=x+2y\\&=(-5)+2\times4\\&=-5+8\\&=3\end{aligned}$$

Explain the reasoning behind each method.

When finding the values of expressions, simplifying the expressions before substituting a number can make the calculation easier.

Q 1 When $x=5$ and $y=-3$, find the values of the following expressions.

(1) $4(x-2y)-(2x-9y)$ (2) $-2x+y-3(x+2y)$

Q 2 When $x=-2$ and $y=\dfrac{1}{3}$, find the values of the following expressions.

(1) $2(3x-6y)+3(5y-2x)$

(2) $(-12x^2y)\div(-4x)$

Try it out
P.25
Enhancement 1-5

Let's Check

1

Monomial Expressions and Polynomial Expressions [P.14] Ex. 1 Q 1

Degrees of Algebraic Expressions [P.15] Ex. 3

Answer the following using ⓐ ~ ⓓ.

ⓐ $\frac{2}{3}x$ ⓑ $5x-4y$ ⓒ $-8x^2$ ⓓ x^2-5x+2

(1) Separate them into monomial and polynomial expressions.

(2) State the terms of the equation for ⓓ.

(3) State their respective degrees.

2

Like Terms [P.16] Ex. 1

Addition of Polynomial Expressions [P.17] Ex. 2

Subtraction of Polynomial Expressions [P.18] Ex. 3

Simplify.

(1) $3x-7y+x+4y$

(2) $2a^2-7a+5+6a^2-1$

(3) $(-5x+6y)+(9x-8y)$

(4) $(x-3y)-(-2x+5y)$

3

Multiplication of Polynomial Expressions and Numbers [P.19] Ex. 4

Division of Polynomial Expressions and Numbers [P.19] Ex. 5

Various Calculations [P.20] Ex. 6

Simplify.

(1) $-3(4x-y+7)$

(2) $(18a-10b)\div 2$

(3) $5(-2a+4b)+3(4a-7b)$

(4) $3(4x-2y)-2(3x+y)$

4

Multiplication and Division of Monomial Expressions [P.21] Ex. 1 Ex. 2 [P.22] Ex. 3 Ex. 4

Simplify.

(1) $(-2a)\times 9b$

(2) $3a\times 5a^2$

(3) $(-6x)^2$

(4) $8ab\div 4a$

(5) $6x^2\div \frac{2}{5}x$

(6) $12xy\div(-6x)\times 2y$

5

Value of Expressions [P.23] Q 1

When $x=-2$ and $y=3$, find the values of the following expressions.

(1) $(x+7y)+(4x-3y)$

(2) $4x^2\times xy\div(-2x)$

Enhancement 1

→ Simplifying Algebraic Expression

Let's use what we have learned for home study and calculation practice.

1 Addition and Subtraction of Polynomial Expressions

(1) $2x+3y+7x+5y$

(2) $-4a+8b-2a-5b$

(3) $5a^2+a^2$

(4) $3x^2-6x+1-2x^2+4x$

(5) $(7a+b)+(-9a+8b)$

(6) $(-3x^2-4x)+(5x^2-x)$

(7) $(8x-6y)-(2x+4y)$

(8) $(-x^2+9x+6)-(7x^2-5x+8)$

(9)
$$\begin{array}{rr} & 2x-6y-5 \\ +) & 3x+2y-4 \\ \hline \end{array}$$

(10)
$$\begin{array}{rr} & -5x+8y \\ -) & 4x-7y \\ \hline \end{array}$$

2 Multiplication and Division of Polynomial Expressions and Numbers

(1) $2(6a-5b+1)$

(2) $(9x-4y)\times(-3)$

(3) $(20a+16b)\div 4$

(4) $(8x-12y)\div(-2)$

3 Various Calculations

(1) $3(a+2b)+6(a-b)$

(2) $-(5x-y)+4(3x-y)$

(3) $2(4x+y)-7x$

(4) $8a-5b-3(a-4b)$

(5) $4(2x-y)-2(x-y+1)$

(6) $\frac{1}{4}(a-3b)-\frac{1}{6}(2a-3b)$

(7) $\frac{2a-b}{6}+\frac{a+b}{8}$

(8) $\frac{4x-y}{3}-\frac{x-3y}{2}$

(9) $x-\frac{x+5y}{2}$

4 Multiplication and Division of Monomial Expressions

(1) $9a\times(-5b)$

(2) $12x\times\frac{5}{6}y$

(3) $3x^2\times 7x$

(4) $(-7a)^2$

(5) $4a\times(-ab)$

(6) $(-18xy)\div(-9x)$

(7) $x^3\div x$

(8) $6x^2\div\frac{3}{4}x$

(9) $x^2\times 4x\div 8xy$

(10) $15a^2b\div(-6ab^2)\times 2ab$

5 Value of Expressions

(1) When $a=-3$ and $b=8$, find the value of a^2-b.

(2) When $x=2$ and $y=-5$, find the value of $8x^2y^3\div 4xy^2$.

(3) When $a=\frac{1}{2}$ and $b=-1$, find the value of $(3a+b)-(a+4b)$.

Answers on P.229

2 Using Algebraic Expression

1 Explaining with Algebraic Expressions

Aim Let's explain the properties of numbers and figures using algebraic equations.

Q Communicate

Find the sum of three consecutive integers such as 6, 7, and 8. Discuss what common properties these sums have.

$6+7+8=\boxed{}$

$10+11+12=\boxed{}$

$23+24+25=\boxed{}$

Mathematical Thinking 2

Using specific numbers, what do you observe about the sum of three consecutive numbers?

Concerning the properties found in **Q**, we cannot check whether they hold true for all numbers just by investigating specific numbers. In such a case, using algebraic expressions allow us to check whether they hold true for all numbers.

Ex. 1 Explain why the sum of three consecutive integers is a multiple of 3 using an algebraic expression.

Mathematical Thinking 3

The sum of 3 consecutive integers being a multiple of 3 can be explained using algebraic expression.

Method Express 3 consecutive integers using a variable and show that their sum is of the form $3\times$(Integer).

Solution

If we let the smallest number be n, the 3 consecutive numbers are expressed as n, $n+1$, $n+2$. Their sum is

$$\begin{aligned} & n+(n+1)+(n+2) \\ =& 3n+3 \\ =& 3(n+1) \end{aligned}$$

Since $n+1$ is an integer, $3(n+1)$ is a multiple of 3.

Therefore, the sum of 3 consecutive integers is a multiple of 3.

Note When we talk about the multiple of a number, the number multiplied by 0 or a negative number is also considered a multiple of that number.

Q 1 From the solution to Ex. 1 on the previous page, what else can we know about the sum of 3 consecutive integers, other than that it is a multiple of 3?

Q 2 Explain Ex. 1 from the previous page by letting n be the middle number.

Q The sum of a two-digit natural number and the number formed by switching the number in the tens place and the number in the units place is a multiple of a certain number. Investigate what multiple the sum becomes.

$$21 + 12 = \square$$
$$35 + 53 = \square$$
$$47 + 74 = \square$$
$$\square + \square = \square$$
$$\square + \square = \square$$

For a two-digit natural number, by letting the number in the tens place be a and the number in the units place be b, we can express it as $10a+b$. Using this, check what you investigated in **Q**.

$$36 = 10 \times 3 + 1 \times 6$$
$$74 = 10 \times 7 + 1 \times 4$$
$$10 \times a + 1 \times b$$

Ex. 2 Using an algebraic expression, explain how the sum of a two-digit natural number and the number formed by switching the number in the tens place and the number in the units place is a multiple of 11.

Solution

If we let the number in the tens place of the two-digit number be a and the number in the units place be b,

The original number is $10a+b$

The number that is formed by switching the digits is $10b+a$

The sum of these two numbers is

$$(10a+b)+(10b+a) = 11a+11b$$
$$= 11(a+b)$$

Since $a+b$ is an integer, $11(a+b)$ is a multiple of 11.

Therefore, the sum of a two-digit natural number and the number formed by switching the number in the tens place and the number in the units place is a multiple of 11.

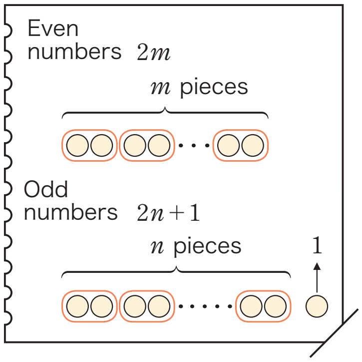
Q 3 What can we say about the difference between a two-digit natural number and the number formed by switching the number in the tens place and the number in the units place? Explain using an algebraic expression.

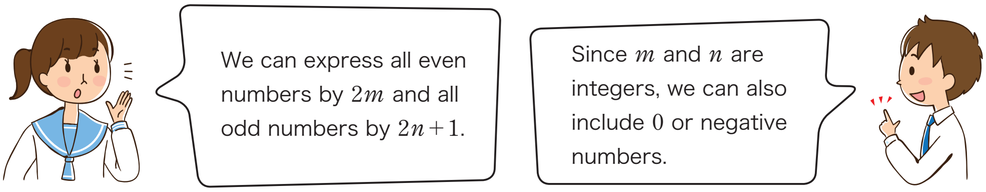
Communicate

Q Out of the sum of the following pairs of numbers, which are odd and which are even?

(1) (Odd) + (Even) (2) (Even) + (Even) (3) (Odd) + (Odd)

Even numbers are divisible by 2, in other words, they are a multiple of 2. Therefore, if we let m be an integer, we can express even numbers as $2m$.
Odd numbers are not divisible by 2, in other words, they are a number 1 larger than an even number. Therefore, if we let n be an integer, we can express odd numbers as $2n+1$.



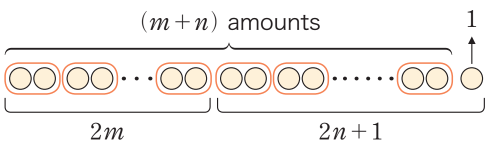


Using this, lets consider **Q**.

1 Yui explained why the sum of an even number and an odd number is odd using the figure on the lower right, as follows.

Yui's Method

If we add an odd number $2n+1$ to an even number $2m$, we get two groups of $(m+n)$ and 1 left over as seen on the right. Therefore, the sum of an even number and an odd number is odd.



Using Yui's method, explain (2) and (3) respectively from **Q**.

2 Takumi explained why the sum of an even number and an odd number is odd using algebraic expressions, as follows. Complete the explanation by filling in the $\boxed{\quad}$ with the appropriate expression or word.

Takumi's Method

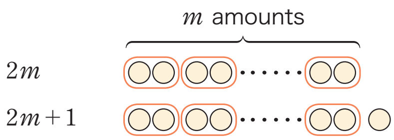
If we let m and n be integers, even numbers are expressed as $2m$, and odd numbers are expressed as $2n+1$. The sum of an even number and an odd number is

$$
\begin{aligned}
&2m+(2n+1)\\
=&2m+2n+1\\
=&2(\boxed{\qquad})+1
\end{aligned}
$$

Since $\boxed{\qquad}$ is an integer, $\boxed{\qquad}$ is an odd number.

Therefore, $\boxed{\qquad\qquad\qquad}$.

3 Yugo explained **1** on the previous page and **2** above, letting even numbers be $2m$ and odd numbers be $2m+1$. Discuss whether Yugo's method is good or not.



4 Using algebraic expressions, write the explanations for (**2**) and (**3**) from **Q** on the previous page. Try explaining to your friends, using the expressions.

5 Looking back on what you learned so far, make conclusions with respect to the following.

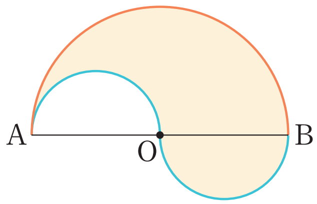
① How can we express "3 consecutive integers", "a two-digit natural number", "even and odd numbers", "a multiple of 3" and so on using variables?

② Why is it better to explain using algebraic expressions?

Q 4 Communicate

In **2** from page 13, explain how we can guess birthdays.

Ex. 3 In the figure on the right, point O is the midpoint of line AB. The sum of the lengths of the arcs of semicircles with diameter AO and BO, respectively, is equal to the length of the arc of a semicircle with diameter AB. Explain this using algebraic expressions.

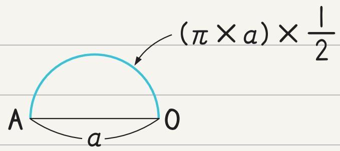


Method Letting AO$=a$, find the respective arc lengths.

Solution

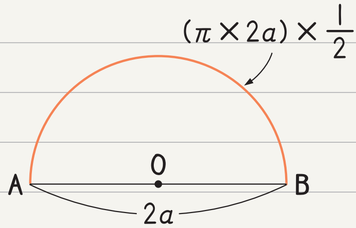
If we let AO$=a$, the length of the arc of a semicircle with diameter AO is

$$(\pi\times a)\times\frac{1}{2}$$



Since point O is the midpoint of line AB,

AO$=$BO



Therefore, the respective lengths of the arcs of semicircles with diameter AO and BO are equal, and their sum is

$$(\pi\times a)\times\frac{1}{2}\times 2=\pi a \qquad ①$$

Also, since AB$=2a$, the sum of the lengths of the arcs of a semicircle with diameter AB is

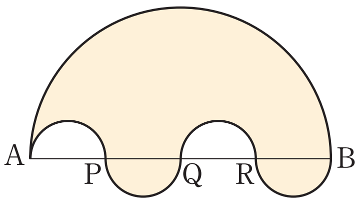
$$(\pi\times 2a)\times\frac{1}{2}=\pi a \qquad ②$$

Since ① and ② are equal, the sum of the lengths of the arcs of the semicircles with diameter AO and BO is equal to the length of the arc of a semicircle with diameter AB.

To make your explanation easier to understand, make sure to draw diagrams.

Write fractions using two lines of your notebook.

Q 5 In the figure on the right, when AP$=$PQ$=$QR$=$RB, explain why the lengths of the arcs of the 4 semicircles with diameters AP, PQ, QR, and RB are equal to the length of the arc of a semicircle with diameter AB, using algebraic expressions.



2 Transformations of Equalities

Aim Let's transform equalities into a form that fits a specific purpose.

Q

The following (1) ~ (3) represent the relationships among distance, speed, and time. Fill in the $\square$ with the appropriate sign.

(1) (Distance) = (Speed) $\square$ (Time)

(2) (Speed) = (Distance) $\square$ (Time)

(3) (Time) = (Distance) $\square$ (Speed)

Depending on what we want to find out, whether distance, speed, or time, as in **Q**, we can transform the expression to show the relationships.

Ex. 1

From the ground up to 11 km above the ground, the air temperature decreases by almost 6 ℃ for each 1 km we go up. If we let the current ground temperature be 18 ℃, and the temperature x km above the ground be y ℃, we can express the relationship between x and y as $y=18-6x$. Change this expression to an expression for finding x.

Solution

Transposing y and $-6x$,	$y=18-6x$
we get	$6x=18-y$
Dividing both sides by 6,	
we get	$x=\dfrac{18-y}{6}$
	Answer $x=\dfrac{18-y}{6}$

$$y=18-6x$$
$$6x=18-y$$

Transforming the equality $y=18-6x$ and deriving $x=\dfrac{18-y}{6}$ such as in Ex. 1 is called **solving for x**.

Note $x=\dfrac{18-y}{6}$ can be written as $x=3-\dfrac{1}{6}y$ or $x=-\dfrac{1}{6}y+3$.

Q 1 In Ex. 1, how many km above the ground is the air temperature 6 ℃ and -30 ℃, respectively?

Q 2 Solve the following equalities for the letter in the 〔 〕.

(1) $x-y=8$ 〔x〕

(2) $y=12-4x$ 〔x〕

(3) $6x+2y=10$ 〔y〕

(4) $3x-y=5$ 〔y〕

Ex. 2 Solve for h in the formula for the area of a triangle $S=\frac{1}{2}ah$.

Solution

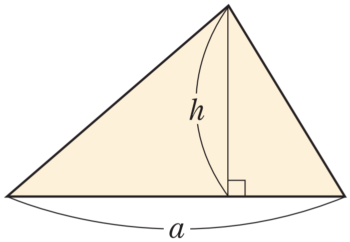
$$S=\frac{1}{2}ah$$

Switching both sides,
we get $\frac{1}{2}ah=S$

Multiplying both sides
by 2, we get $ah=2S$

Dividing both sides
by a, we get $h=\frac{2S}{a}$

Answer $h=\frac{2S}{a}$

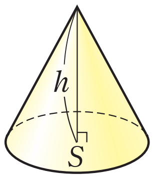


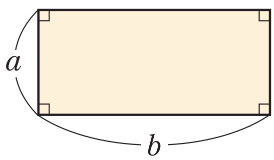
We switch both sides of the expression so that it is easier to solve for h.

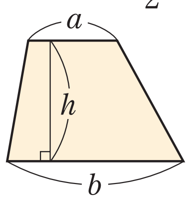
Q 3 Using the expression you found in Ex. 2, find the height of a triangle with area 42 cm^2 and base 12 cm.

Q 4 Solve the following equalities for the variable in the 〔 〕.

(1) $V=\frac{1}{3}Sh$ 〔h〕 (2) $\ell=2(a+b)$ 〔a〕 (3) $S=\frac{(a+b)h}{2}$ 〔a〕







Let's Check

2 Using Algebraic Expression

1 Explaining with Algebraic Expressions
[P.26] Ex. 1
[P.29] 2

Answer the following regarding two consecutive odd numbers, such as 5 and 7.

(1) Letting n be an integer, if we let the smaller odd number be $2n+1$, how can we express the larger odd number?

(2) Explain why the sum of these two consecutive odd numbers is a multiple of 4.

2 Transformations of Expressions
[P.31] Q 2
[P.32] Ex. 2

Solve the following equalities for the variable in the 〔 〕.

(1) $4x-y=8$ 〔x〕 (2) $m=\frac{a+b}{2}$ 〔a〕

Chapter 1 Summary Problems

Answers on P.230, 231

Key Ideas

1 Answer the questions below using ⓐ ~ ⓕ .

ⓐ	$4x+7$	ⓑ	$2x^2$	ⓒ	$3x-5y$
ⓓ	$-8x$	ⓔ	$6xy+9y$	ⓕ	x^2-6x+1

(1) Which are monomial expressions?
(2) Which are linear expressions?

2 Simplify.

(1) $8a^2+6a+a^2-2a$　(2) $-2x-8y+7y-3x+5$

(3) $(4a-9b)+(3a+5b)$　(4) $(5x+2y)-(6x-4y)$

3 Simplify.

(1) $(20x-4y)\div(-4)$　(2) $(5a-8b)+3(-a+2b)$

(3) $5(x+3y)-4(2x-y)$　(4) $\dfrac{3x+y}{4}-\dfrac{x-y}{6}$

(5) $7x\times4y$　(6) $3a^2\times(-2a)$

(7) $(-9x)^2$　(8) $(-16a^2)\div4a$

(9) $6xy\div\dfrac{3}{7}x$　(10) $4x^2\div6x^2\times3x$

4 Correct the mistakes in the following calculations and find the answers.

(1) $18xy\div3x\times2y$
$=18xy\div6xy$
$=3$

(2) $6ab\div\left(-\dfrac{2}{3}a\right)$
$=6ab\times\left(-\dfrac{3}{2}a\right)$
$=-9a^2b$

5 When $x=6$ and $y=-5$, find the values of the following expressions.

(1) $14xy^2\div7y$　(2) $(3x+5y)-(x+6y)$

6 Explain why the sum of 3 integers, with a difference of 3 such as 1, 4, 7, is a multiple of 3 using algebraic expressions.

7 Solve the following for the variable in the 〔 〕.

(1) $3x+2y=10$ 〔y〕 (2) $a=\dfrac{4b+3c}{7}$ 〔c〕

Application

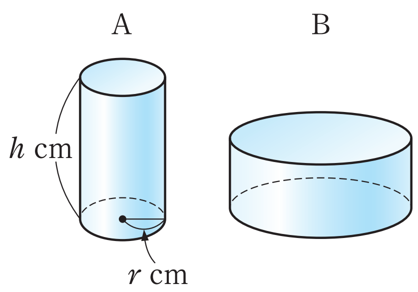
1 Simplify.

(1) $\dfrac{1}{2}x+y-\left(\dfrac{2}{3}x-\dfrac{y}{2}\right)$ (2) $x-y-\dfrac{3x-y}{4}$

(3) $3a^2\div 6ab\times(-2a)^2$ (4) $9x^2\times(-xy)\div\dfrac{3}{5}y^3$

2 If we let $A=x^2-3x-5$ and $B=-2x^2+x+7$, what expression do we need to subtract from A to obtain a difference of B?

3 Cylinder A has a base radius of r cm and a height of h cm. Cylinder B has a base radius twice that of cylinder A, and a height $\dfrac{1}{2}$ that of cylinder A. Use algebraic expressions to explain how many times larger the volume of cylinder B will be than cylinder A.



4 In the calendar on the right, the sum of the 3 numbers 2, 9, and 16 marked by ▯ is equal to 3 times the middle number, 9. Can we say the same thing about the sum of 3 numbers aligned vertically for other places on the calendar, too? Explain using algebraic expressions.

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Practical Use

1 Yui is investigating the difference between a three-digit natural number and the number formed by switching the digit in the hundreds place and the digit in the units place.

For 524, $524-425=99$
For 937, $937-739=198$
For 259, $259-952=-693$

From these results, Yui predicted the following, and explained her reasoning below. Complete Yui's explanation.

Yui's Prediction

The difference between a three-digit natural number and the number formed by switching the digit in the hundreds place and the digit in the units place is a multiple of 99.

If we let the digit in the hundreds place be a, the digit in the tens place be b, and the digit in the units place be c, the three-digit natural number is expressed by $\boxed{}$.
The natural number with the hundreds place and the units place switched is expressed by $\boxed{}$.The difference between the two numbers is

$\boxed{}$

Therefore, the difference between a three-digit natural number and the number formed by switching the digit in the hundreds place and the digit in the units place is a multiple of 99.

2 From the expression in Yui's explanation, there are also things we can know other than "the difference between the two numbers is a multiple of 99". From the following ⓐ ~ ⓕ, choose all that apply.

ⓐ The difference between the two numbers is a multiple of 6.
ⓑ The difference between the two numbers is a multiple of 11.
ⓒ The difference between the two numbers is an odd number.
ⓓ The difference between the two numbers is an even number.
ⓔ The difference between the two numbers does not have any relevance to the tens place of the original number.
ⓕ The difference between the two numbers is 99 times the difference after subtracting the digit in the units place from the digit in the hundreds place.

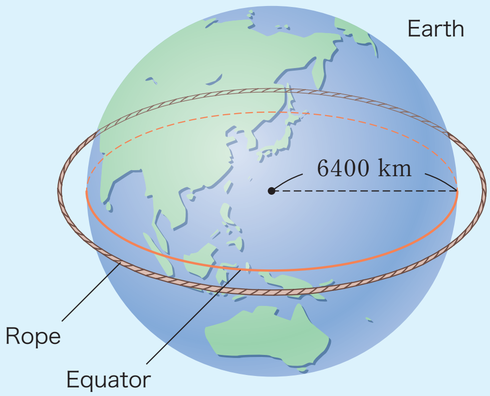
3 So far, we have learned that
"The difference between a two-digit natural number and the number formed by switching the digit in the tens place and the digit in the units place is a multiple of 9" and
"The difference between a three-digit natural number and the number formed by switching the digit in the hundreds place and the digit in the units place is a multiple of 99."
From this, Daiki predicted that
"The difference between a four-digit natural number and the number formed by switching the digit in the thousands place and the digit in the units place is a multiple of 999."
Is this correct? If you think it is correct, explain using algebraic expressions. If you think it is incorrect, give an example where the difference is not a multiple of 999.

Deep Learning

What if We Tie a Rope Around the Equator?

The radius of the Earth is about 6400 km. A rope 10 m longer than the length of the Earth's equator forms a circle in the air above the equator.

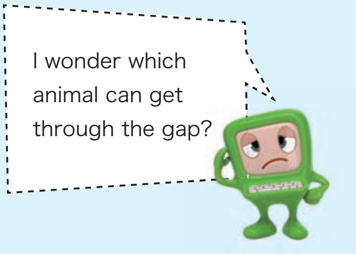
In the above scenario, which of the following animals can get through the gap between the rope and the equator?



ⓐ Mouse (Height 5 cm)
ⓑ Cow (Height 1 m 50 cm)
ⓒ Elephant (Height 3 m)

1 If we let the radius of the Earth be r m, the length of the equator is $2\pi r$ m. Express the length of the rope and the radius of the circle formed by the rope using algebraic expressions.

2 Find the difference between the radius of the circle formed by the rope and (the radius of) the Earth. If we let $\pi = 3.14$, what is the difference?



The result of **2** is not related to the radius r. Therefore, in the problem above, we will get the same result even if we replace the Earth with the moon or a gas tank.

▶ We are construcing an athletics track. The curved section of the track is a semicircle. For going around the track once, how many m do we need to adjust the starting line of adjacent lanes? Let the width of each lane be 1 m, and π be 3.14.