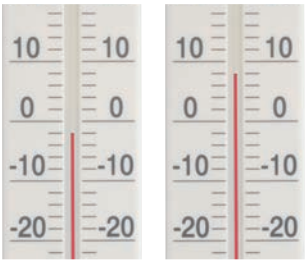
1 Positive and Negative Numbers

1 Numbers with Signs

• Aim • Let's investigate how the numbers with "−" are used.

Quantities with 0 as the Reference Point

Q The thermometers on the right show the temperatures at 6:00 AM in Niigata and Kagoshima, respectively. What temperature does each show? Consider how much higher or lower than 0 ℃ they are.



Niigata Kagoshima

If the temperature is 2 ℃ lower than 0 ℃, we use a − sign and write it as −2 ℃. We read it as **"negative 2 ℃"**. If the temperature is 8 ℃ higher than 0 ℃, we use a + sign and write it as +8 ℃. We read it as **"positive 8 ℃"**.

When + and − are used in this way, they are called the **positive sign** and **negative sign**, respectively.

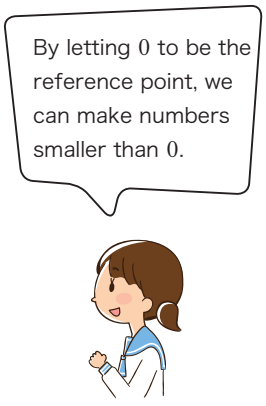
Q 1 Express the following temperatures with a positive or negative sign.

(1) The temperature that is 6.5 ℃ higher than 0 ℃

(2) The temperature that is 10 ℃ lower than 0 ℃

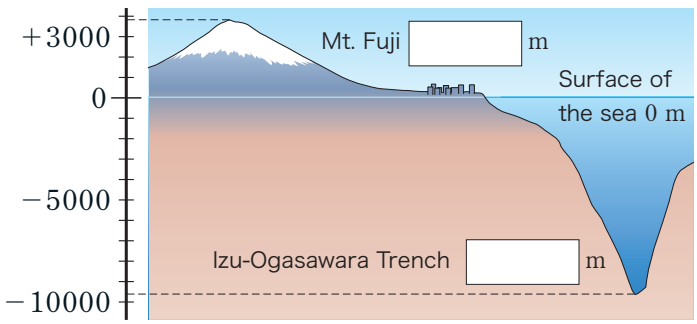
If we let 0 ℃ be the reference temperature at which water freezes and ice melts, we can express temperatures higher than 0 ℃ with a positive sign and temperatures lower than 0 ℃ with a negative sign.

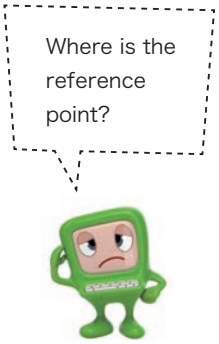
Aside from temperature, various quantities can be represented using positive and negative signs by setting a reference point and letting it be 0.



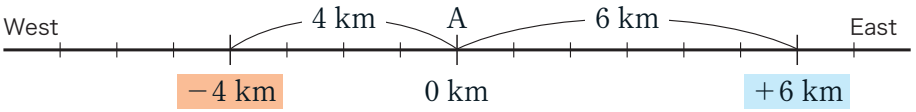
Various Quantities with "−"

The peak of Mt. Fuji is **3776 m** above sea level and the deepest part of the Izu-Ogasawara Trench is **9780 m** below sea level. How can we express these values on the following figure with a set reference point using the positive and negative signs?





Ex. 1 If we let point A be the reference point 0 km and express the point "6 km to the east from A" as +6 km, we can express the point " 4 km to the west from A" as −4 km.



Q 2 In Ex. 1, what points do −7 km and +2.5 km represent on the number line? Indicate the point with an arrow ↑. Express them using words.

Q 3 Express the following quantities using a positive or a negative sign.

(1) "500 yen loss", when "400 yen profit" is expressed as +400 yen.

(2) "30 minutes from now", when "20 minutes before now" is expressed as −20 minutes.

(3) "4 ℃ lower than yesterday's maximum temperature", with respect to today's maximum temperature, when "3 ℃ higher than yesterday's maximum temperature" is expressed as +3 ℃ .

Q 4 When records in track and field are shown such as in the 100 m sprint, a tailwind of 0.9 m per second is displayed as "+0.9 m/s." What does "−2.3 m/s" mean?

Positive and Negative Numbers

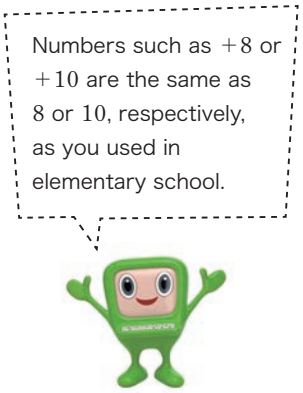
Express the following numbers using a positive or a negative sign.

(1) The number 8 larger than 0

(2) The number 4 smaller than 0

Numbers larger than 0 such as $+8$, $+10$, and so on are called **positive numbers**. Numbers smaller than 0 such as -4, -9, and so on are called **negative numbers**.

0 is neither a positive nor a negative number.

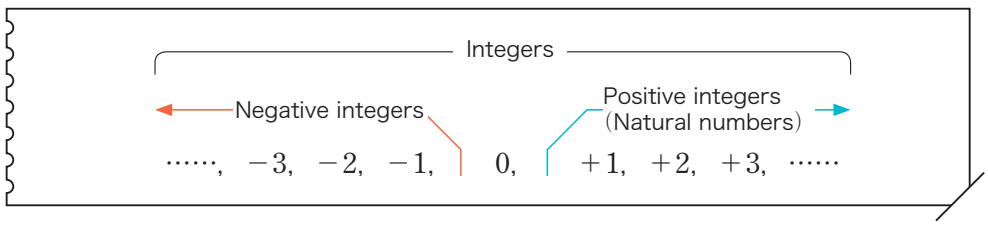


Q 5 Which of the following numbers are positive and which are negative? State how much larger or smaller than 0 they are.

(1) -6 (2) $+3$ (3) $+1.2$ (4) $-\frac{2}{5}$ (5) -0.1

In elementary school, we learned about positive numbers and 0. In junior high school, we will also include negative numbers. From now on, integers will include positive numbers, 0, and negative numbers.

Positive integers are also called **natural numbers**.



So when we set the reference point as 0, we can express numbers larger and smaller than 0 using a positive and a negative sign, respectively.

In elementary school, we used to express numbers on a number line. Can we represent negative numbers on a number line, too?

P.17

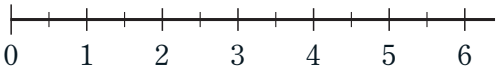
2 Comparing Numbers

• Aim • Let's represent the positive and negative numbers on the number line and compare their values.

Number Line with Negative Numbers

Q

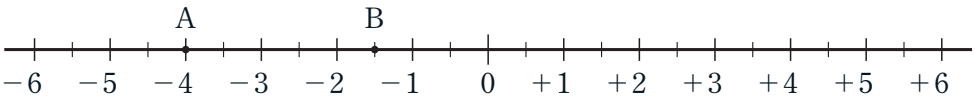
(1) Mark the points corresponding to 2, 3.5 and $\frac{1}{2}$ on the number line below. Compare their values.



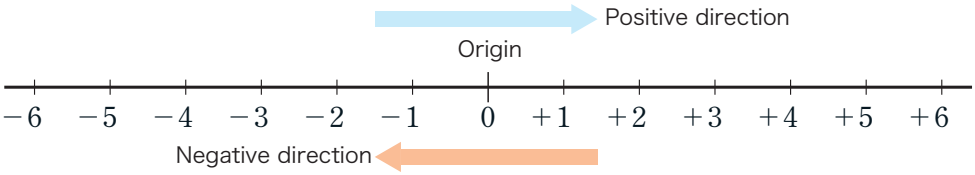
(2) What do we need to do with the number line to represent negative numbers on it? Use the number line above to write your answer.

We can write negative numbers on the number line in (1) from Q by extending the number line from 0 to the left, marking ticks using the same interval as the original number line, and matching the numbers corresponding to the points on the number line.

On the following number line, points A and B correspond to -4 and -1.5, respectively.



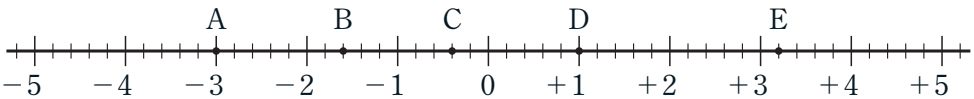
The point corresponding to 0 is called the **origin**. The direction to the right is called the positive direction and the direction to the left is called the negative direction.



Q 1 Draw a number line, and mark the points that correspond to the following numbers.

$+4$, $+0.5$, -2, -5, -3.5, $-\frac{3}{2}$

Q 2 State the numbers corresponding to the points A, B, C, D, and E.



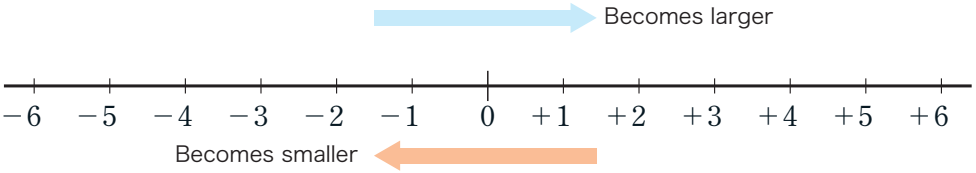
Comparing Numbers Using the Number Line

Q Communicate

Which is larger, -2 or -5?
Explain using the number line.

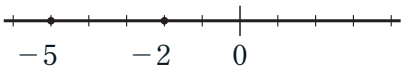
Remember that in positive numbers, those farther to the right on the number line are larger, while those farther to the left are smaller.

In the range of negative numbers, those farther to the right on the number line are larger, while those farther to the left are smaller, the same as for the positive numbers.



Ex. 1

For -2 and -5, -2 is to the right of -5 on the number line. Thus, -2 is larger than -5. We can represent this using an inequality sign as $-5<-2$ or $-2>-5$



Ex. 2

We can represent $-2, +3$ and -4 on the number line as shown in the figure on the right. Expressing this using an inequality sign, from smallest to largest,
$-4<-2<+3$
from largest to smallest,
$+3>-2>-4$

Let's think about why we cannot express it as $-2<+3>-4$.

Q 3

Compare the following pairs of numbers using inequality signs.

(1) $+3,\ +4$ (2) $-4,\ -6$

(3) $+0.1,\ -0.2$ (4) $-\frac{2}{3},\ -\frac{1}{3}$

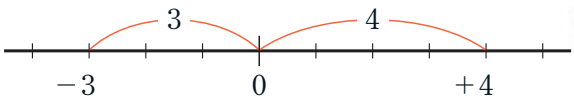
(5) $+1,\ -3,\ 0$ (6) $-2,\ +5,\ -5$

Absolute Values

Q

When we represent $+4$ and $+6$ on the number line, which number is farther away from the origin?

The distance between the point that corresponds to a given number and the origin is called the **absolute value**. For example, the absolute value of $+4$ is 4, while the absolute value of -3 is 3. The absolute value of 0 is 0.

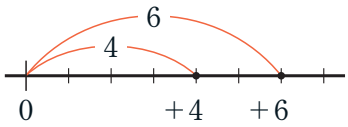


You can also think of absolute value as simply the number itself without its + or − sign.

Q 4 State the absolute values of -7 and $+5.2$, respectively.

Q 5 State the numbers with an absolute value of 10 and $\frac{2}{3}$, respectively.

When we compare the two positive numbers $+4$ and $+6$, the absolute value of $+6$ is larger. On the number line, $+6$ is farther to the right.



Thus, for two positive numbers, the one with the larger absolute value is larger.

Q 6 Communicate

When we compare the absolute values of two negative numbers, what can we say about their values? Explain using an example.

Comparing the values of two numbers can be summarized as follows.

IMPORTANT

Comparing Two Numbers

1. **Positive numbers are larger than 0, negative numbers are smaller than 0. Positive numbers are larger than negative numbers.**
2. **When there are two positive numbers, the one with the larger absolute value is larger.**
3. **When there are two negative numbers, the one with the larger absolute value is smaller.**

Now we know about positive and negative numbers. When we learned about new numbers in elementary school, we also learned how to calculate using them.

Can we add using positive and negative numbers such as $(+5)+(-3)$? P.21

Let's Check

1 Various Quantities with "−" [P.15] Q 3

Answer the following, using positive and negative signs to express quantities.

(1) If we let point A be the reference point 0 km, and express the point "3 km to the north from A" as +3 km, how can we express the point "5 km to the south of A"?

(2) When "200 yen loss" is expressed as −200 yen, what does +300 yen express?

2 Positive and Negative Numbers [P.16] Q 5

Answer the following.

$$-12,\quad +7,\quad 0,\quad +0.6,\quad -3,\quad +25,\quad -\frac{8}{3}$$

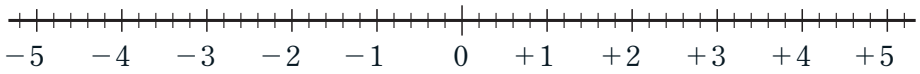
(1) Which are positive numbers? Which are negative numbers?

(2) Which are integers? Which are natural numbers?

3 Number Line with Negative Numbers [P.17] Q 1

Mark the points that correspond to the following numbers on the number line below.

$$-5,\quad +3,\quad -2.8,\quad +\frac{3}{5}$$



4 Comparing Numbers Using the Number Line [P.18] Ex.1 Ex.2

Compare the following pairs of numbers using the inequality signs.

(1) −3, +5

(2) 0, −7

(3) −1.6, −2.4

(4) +1, −3, −2

5 Absolute Values [P.19] Q 4 Q 5

State the absolute values of +16 and $-\frac{9}{7}$, respectively. State the numbers with absolute values of 9 and 0, respectively.

2 Addition and Subtraction

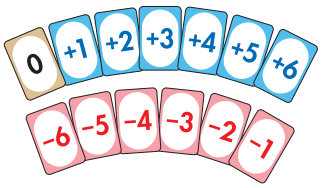
1 Addition

• Aim • Let's consider addition of positive and negative numbers using a card game.

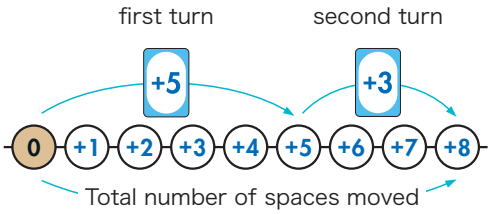
Q Let's play the card game from Appendix ①.
Rules: Shuffle a deck of 13 cards like the one shown below, and place them in a stack face down. Position the players' pieces on the starting point 0. The players take turns drawing a card from the deck, and moving their own piece according to the number shown. The player whose piece reaches the goal first wins.

[How to move the pieces]

When a **+2** is drawn, move 2 spaces toward the goal.
When a **−3** is drawn, move 3 spaces away from the goal.
When a **0** is drawn, don't move the piece.



In the card game from **Q**, if we let a certain player draw a $+5$ on the first turn and a $+3$ on the second turn, the total number of spaces moved is $+8$.



We can express this using the following addition math sentence.

$$\underset{\substack{\vdots\\ \text{(number of spaces moved on first turn)}}}{(+5)} \overset{\substack{\\ \text{plus}}}{+} \underset{\substack{\vdots\\ \text{(number of spaces moved on second turn)}}}{(+3)} = \underset{\substack{\vdots\\ \text{(total number of spaces moved)}}}{+8}$$

Q 1 Using the card game from **Q**, fill in the following table with the addition math sentence to find the total number of spaces moved.

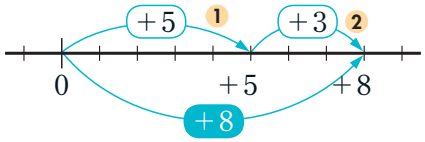
	Number of spaces moved on first turn	Number of spaces moved on second turn	Addition math sentence to find the total number of spaces moved	Total number of spaces moved
ⓐ	-5	-3		?
ⓑ	$+5$	-3		?
ⓒ	-5	$+3$		?

Let's consider the value of addition math expressions which we set up on the previous page using the number line.

Adding Two Numbers with the Same Sign

Ex. 1

(1) $(+5)+(+3)$

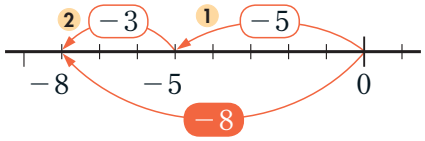


$(+5)+(+3)=+8$

① Move 5 spaces in the positive direction from 0.
② Move 3 spaces in the positive direction.

Thus, the total number of spaces moved is $+8$.

(2) $(-5)+(-3)$



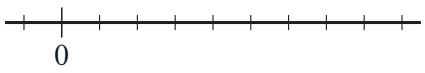
$(-5)+(-3)=-8$

① Move 5 spaces in the negative direction from 0.
② Move 3 spaces in the negative direction.

Thus, the total number of spaces moved is -8.

Q 2 Calculate using the number line.

(1) $(+3)+(+4)$

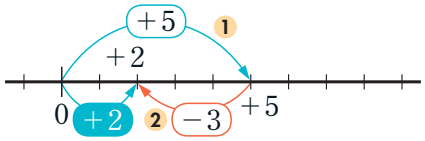


(2) $(-2)+(-6)$

Adding Two Numbers with Different Signs

Ex. 2

$(+5)+(-3)$

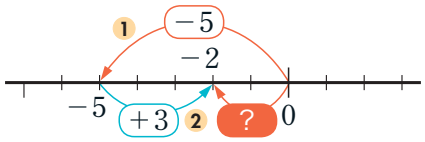


$(+5)+(-3)=+2$

① Move 5 spaces in the positive direction from 0.
② Move 3 spaces in the negative direction.

Thus, the total number of spaces moved is $+2$.

Q 3 We will explain the calculation of $(-5)+(+3)$ using the number line. Fill in the ☐ with the appropriate number or word.



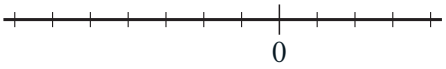
$(-5)+(+3)=$ ☐

① Move ☐ spaces in the negative direction from 0.
② Move 3 spaces in the ☐ direction.

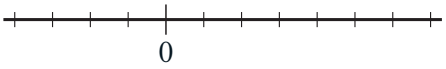
Thus, the total number of spaces moved is ☐.

Q 4 Calculate using the number line.

(1) $(+2)+(-6)$



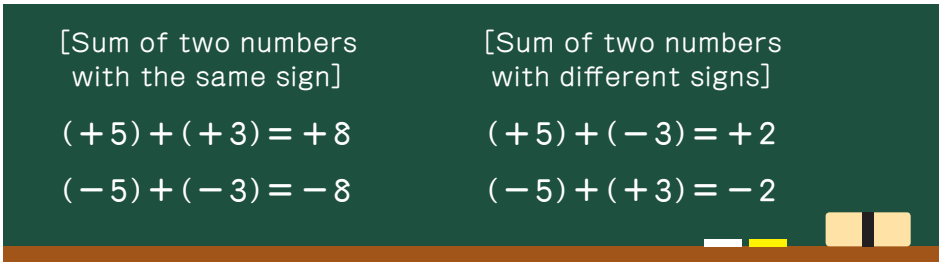
(2) $(-2)+(+7)$



The operation of adding positive and negative numbers is also called **addition**. The result of addition is called the sum.

Addition Using Signs and Absolute Values

From what we have learned from the sum of two numbers with the same sign and the sum of two numbers with different signs, let's discuss what we have noticed about signs and absolute values.



Let's find the sum of two numbers by using Q.

Ex. 3 Sum of two numbers with the same sign

(1) $(+9)+(+3)$
$=+(9+3)$
$=+12$

(2) $(-18)+(-5)$
$=-(18+5)$
$=-23$

Ex. 4 Sum of two numbers with different signs

(1) $(+27)+(-12)$
$=+(27-12)$
$=+15$

(2) $(-10)+(+6)$
$=-(10-6)$
$=-4$

Q 5 Calculate.

(1) $(+4)+(+13)$

(2) $(-8)+(-16)$

(3) $(-7)+(+8)$

(4) $(+14)+(-19)$

Q 6 Find the sum of $+3$ and -3.

The addition of positive and negative numbers can be summarized as follows.

IMPORTANT **Addition of Positive and Negative Numbers**

1. Sum of two numbers with the same sign
   - Sign: same as the two numbers
   - Absolute value: sum of the absolute values of the two numbers
2. Sum of two numbers with different signs
   - Sign: the sign of the number with the larger absolute value
   - Absolute value: the difference between the absolute values of the larger number and smaller number

The sum of two numbers with different signs that have the same absolute value is 0.

No matter to what number we add 0, the sum will always be equal to the number to which 0 is added, for example $(+3)+0=+3$. Similarly, no matter what number we add to 0, the sum will always be equal to the number being added, for example $0+(-2)=-2$.

Q 7 Calculate.

(1) $(+9)+(+5)$ (2) $(-5)+(-7)$ (3) $(+8)+(-3)$

(4) $(-25)+(+16)$ (5) $(-21)+(+21)$ (6) $0+(-37)$

Addition of Decimals and Fractions

Ex. 5

(1) $(-1.2)+(-0.5)$

$=-(1.2+0.5)$

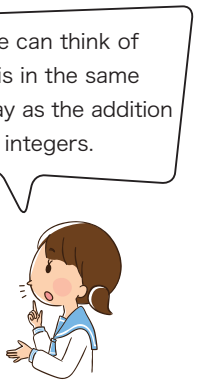
$=-1.7$

(2) $\left(+\frac{1}{2}\right)+\left(-\frac{2}{3}\right)$

$=\left(+\frac{3}{6}\right)+\left(-\frac{4}{6}\right)$

$=-\left(\frac{4}{6}-\frac{3}{6}\right)$

$=-\frac{1}{6}$



Q 8 Calculate.

(1) $(+0.3)+(+1.2)$ (2) $(-0.7)+(+0.5)$

(3) $(+1.4)+(-0.9)$ (4) $\left(-\frac{3}{5}\right)+\left(+\frac{4}{5}\right)$

(5) $\left(-\frac{1}{2}\right)+\left(-\frac{3}{4}\right)$ (6) $\left(+\frac{1}{4}\right)+\left(-\frac{5}{6}\right)$

Try it out

P.35 Enhancement 1-1

Commutative and Associative Properties of Addition

Q

Do the rules for addition that we learned in elementary school still hold true for addition of positive and negative numbers? Calculate the following ⓐ and ⓑ and compare the results. Check using other numbers.

(1) ⓐ $(+5)+(-7)$

ⓑ $(-7)+(+5)$

(2) ⓐ $\{(-3)+(+6)\}+(-4)$

ⓑ $(-3)+\{(+6)+(-4)\}$

Review

Changing the order of the addend and augend will not change the sum.

$\square + \triangle = \triangle + \square$

When adding three numbers, changing the order of addition will not change the sum.

$(\square + \triangle) + \bigcirc$
$= \square + (\triangle + \bigcirc)$

Elementary 4

Mathematical Thinking 2

You can find the rules for addition by looking at the results of several addition math expressions.

Note For double parentheses, you can use the symbols { }.

The following also hold true for addition of positive and negative numbers.

Commutative property of addition

$$a+b=b+a$$

Associative property of addition

$$(a+b)+c=a+(b+c)$$

The letters can be replaced by any number, including positive numbers, negative numbers, and 0.

For addition, we can use the commutative property and associative property to change the order of numbers and order of calculation.

Ex. 6

$(+11)+(-5)+(+9)+(-7)$
$=(+11)+(+9)+(-5)+(-7)$ — Change the order of the numbers using the commutative property.
$=(+20)+(-12)$ — Find the sum of positive numbers and negative numbers using the associative property.
$=+8$

Q 9

Calculate.

(1) $(-12)+(+7)+(-6)+(+3)$

(2) $(+19)+(-5)+(-28)+(-14)$

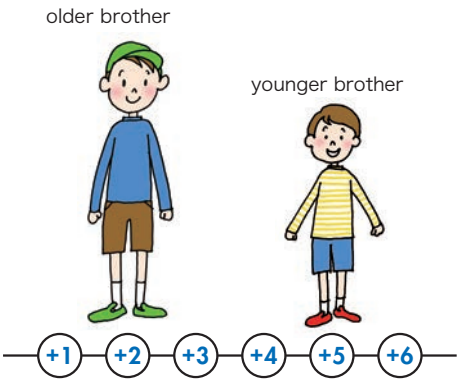
We can now calculate addition of positive and negative numbers just like how we did calculations in elementary school.

Can we do division of positive and negative numbers too? P.26

2 Subtraction

• Aim • Let's consider the subtraction of positive and negative numbers using a card game.

Q In the card game from Appendix ①, two brothers take turns moving their pieces. The older brother moved $+2$, and the younger brother moved $+5$. On the next turn, how many spaces and in which direction does the older brother have to move in order to overtake the younger brother?



In the card game from Q, if we let the older brother overtake the younger by moving $\square$ spaces on his second turn, we can come up with the following addition math sentence.

$$\underset{\substack{\vdots\\ \text{(number of spaces moved on first turn)}}}{(+2)} + \underset{\substack{\vdots\\ \text{(number of spaces moved on second turn)}}}{(\ \square\ )} = \underset{\substack{\vdots\\ \text{(total number of spaces moved)}}}{+5}$$

Thus, in order to find the appropriate number $\square$, we can consider a math sentence such as the following.

Mathematical Thinking 1

We can still consider the subtraction of positive and negative numbers as the opposite of addition.

$$\underset{\substack{\vdots\\ \text{(total number of spaces moved)}}}{(+5)} \underset{\text{minus}}{-} \underset{\substack{\vdots\\ \text{(number of spaces moved on first turn)}}}{(+2)} = \underset{\substack{\vdots\\ \text{(number of spaces moved on second turn)}}}{\square}$$

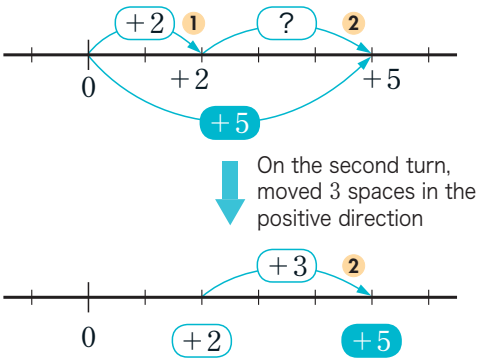
Q 1 For the card game from Q, fill in the following table with the subtraction sentence to find the amount moved on the second turn.

	Amount moved on first turn	Amount moved on second turn	Total amount moved	Subtraction sentence to find the amount moved on the second turn
ⓐ	-3	?	$+2$	
ⓑ	$+4$	?	$+1$	
ⓒ	-2	?	-6	

Let's consider the calculations of the subtraction sentences we set up on the previous page using the number line.

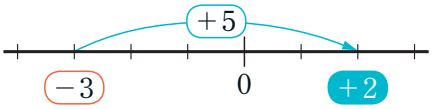
If we consider subtracting one number from another on the number line, $+5$ is 3 spaces in the positive direction from $+2$. Therefore, we know that the amount moved on the second turn is $+3$. From this, we get the following.

$(+5)-(+2)=+3$



Ex. 1

$(+2)-(-3)$

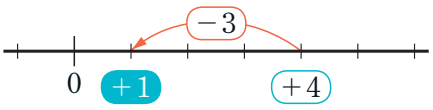


$(+2)-(-3)=+5$

$+2$ is 5 spaces in the positive direction from -3. Thus, the number of spaces moved on the second turn is $+5$.

Ex. 2

$(+1)-(+4)$



$(+1)-(+4)=-3$

$+1$ is 3 spaces in the negative direction from $+4$. Thus, the number of spaces moved on the second turn is -3.

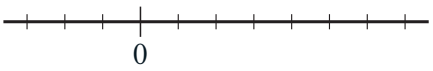
Q 2 Communicate

Explain the calculation for $(-6)-(-2)$ using the number line.

Q 3

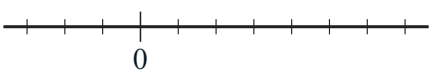
Calculate using the number line.

(1) $(+2)-(+4)$

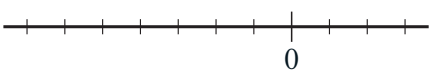


(2) $(+3)-(-6)$

(3) $(-1)-(+3)$



(4) $(-4)-(-5)$



The operation of subtracting positive and negative numbers is also called **subtraction**. The result of subtraction is called the difference.

Relationship between Addition and Subtraction

Q Communicate

For the following subtraction problems (1) ~ (4), choose from the addition expressions ① ~ ④ on the right that give the same answer and fill in the ☐. From the results, discuss what you observed.

(1) $(+3)-(+5)=$ ☐
(2) $(+3)-(-5)=$ ☐
(3) $(-3)-(+5)=$ ☐
(4) $(-3)-(-5)=$ ☐

① $(+3)+(+5)$
② $(+3)+(-5)$
③ $(-3)+(+5)$
④ $(-3)+(-5)$

From Q, the following can be said:

"subtracting $+5$" is the same as "adding -5".
"subtracting -5" is the same as "adding $+5$".

Ex. 3

(1) $(+6)-(+9)$
$=(+6)+(-9)$
$=-3$

(2) $(+6)-(-9)$
$=(+6)+(+9)$
$=+15$

(3) $(-4)-(+10)$
$=(-4)+(-10)$
$=-14$

(4) $(-4)-(-10)$
$=(-4)+(+10)$
$=+6$

Q 4

Change the following subtraction math expressions into addition math sentences and then calculate.

(1) $(+5)-(+12)$
(2) $(+3)-(-8)$
(3) $(-15)-(+10)$
(4) $(-7)-(-7)$

Subtraction of positive and negative numbers can be summarized as follows.

IMPORTANT **Subtraction of Positive and Negative Numbers**

The subtraction of positive and negative numbers is changing the sign of the number being subtracted and then adding it.

Q 5 Calculate.

(1) $0-(+3)$ (2) $0-(-5)$

Subtracting a number from 0 is the same as changing the sign of the number.
When subtracting 0 from a number, the difference is always equal to that number.

$(+8)-0=+8,\quad (-1)-0=-1$

Q 6 Calculate.

(1) $(+8)-(+2)$ (2) $(+3)-(+7)$ (3) $(+5)-(-4)$

(4) $(-12)-(+9)$ (5) $(-27)-(-15)$ (6) $(-16)-(-16)$

(7) $(+38)-(-12)$ (8) $(-10)-0$ (9) $0-(-24)$

Q 7 For the daily maximum temperatures around Japan on pages 12 and 13, answer the following questions.

(1) Construct a math expression to find the previous day's temperature in Sapporo, and find the answer.

(2) Do the same as (1) for the previous day's temperature at Sendai.

Subtraction of Decimals and Fractions

Ex. 4

(1) $(+3.2)-(-1.8)$
$=(+3.2)+(+1.8)$
$=+5$

(2) $\left(-\frac{1}{2}\right)-\left(+\frac{1}{3}\right)$
$=\left(-\frac{1}{2}\right)+\left(-\frac{1}{3}\right)$
$=\left(-\frac{3}{6}\right)+\left(-\frac{2}{6}\right)$
$=-\frac{5}{6}$

Q 8 Calculate.

(1) $(-2.7)-(-3.4)$ (2) $(-1)-(+0.8)$

(3) $\left(+\frac{1}{5}\right)-\left(-\frac{4}{5}\right)$ (4) $\left(-\frac{3}{4}\right)-\left(+\frac{1}{2}\right)$

(5) $(-0.75)-\left(-\frac{3}{4}\right)$ (6) $\left(-\frac{7}{4}\right)-(+0.4)$

Try it out

P.35
Enhancement 1-2

Let me ask!

Do the commutative property and associative property also hold true for subtraction?

P.34

Try

The Kyushu Shinkansen stops at a total of 6 stations between Kagoshima Chuo Station and Kumamoto Station. The following table shows the distances between each station, letting Kagoshima Chuo Station be the reference point 0 km, and the direction of Kumamoto station be the positive direction.

Station	Kagoshima Chuo	Sendai	Izumi	Shinminamata	Shinyatsusiro	Kumamoto
Distance (km)	0	+46	+79	+95	+138	+171

If we let Izumi Station be the reference point, how can we express the distances between each station using positive and negative numbers? Fill in the following table with appropriate numbers.

Station	Kagoshima Chuo	Sendai	Izumi	Shinminamata	Shinyatsusiro	Kumamoto
Distance (km)			0	+16		

We can now subtract positive and negative numbers by changing them into addition math expression.

Even if addition and subtraction are combined such as in $(+2)+(-5)-(-4)$, I think I can now calculate by changing it into an addition math expression.

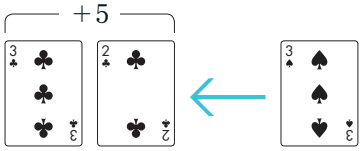
P.31

close up

Calculating Using Cards

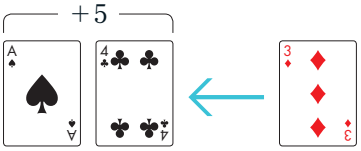
I played a game of cards, letting the black cards be positive points, and the red cards be negative points. What will be my final total score for each of the following four rounds?

1 I had +5 in my hand, and I drew a black 3.



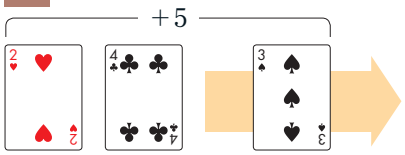
$(+5)+(+3)=$ ☐

2 I had +5 in my hand, and I drew a red 3.



$(+5)+(-3)=$ ☐

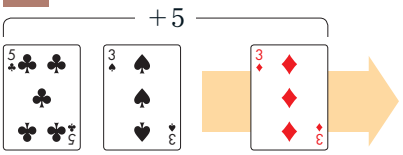
3 I had +5 in my hand :



The black 3 was taken away.

$(+5)-(+3)=$ ☐

4 I had +5 in my hand :



The red 3 was taken away.

$(+5)-(-3)=$ ☐

Decide on the rules of the game and try it yourself.

3 Calculation with Both Addition and Subtraction

• Aim • Let's consider calculations with both addition and subtraction.

Q For the following math expressions which contain both addition and subtraction, can you come up with a way to find the answer?

(1) $(+2)+(-5)-(-4)$ (2) $(-6)-(+7)-(-6)$

We can change math expressions with both addition and subtraction, as shown on the right, to addition-only.

$$(+2)+(-5)-(-4)$$
$$=(+2)+(-5)+(+4)$$

From the addition math expression $(+2)+(-5)+(+4)$, the numbers $+2$, -5 and $+4$ which are joined together by the addition sign $+$ are called the **terms** of this math expression.

$+2$ and $+4$ are **positive terms**, and -5 is a **negative term**.

$$\overbrace{(+2)+(-5)+(+4)}^{\text{Positive terms}}$$
$$\underbrace{}_{\text{Negative term}}$$

Q 1 Change the following into addition-only math expressions. List the positive terms and negative terms, respectively.

(1) $(+4)-(-3)$ (2) $(+7)-(+2)$

(3) $(-9)+(-4)-(-6)$ (4) $(-5)-(-3)-(-8)$

We can write addition-only math expressions without the addition sign $+$ and the parentheses. Also, if the first term of the math expression is positive, we can remove the positive sign $+$.

$$(+2)+(-5)+(+4)$$
$$=2-5+4$$

Q 2 Change the following into addition-only math expressions, remove the parentheses, and line up the terms.

(1) $(+10)-(+15)$ (2) $(-7)-(-9)$

(3) $(-1)+(-4)-(-7)$ (4) $(+6)-(-8)-(+16)$

(5) $(+7)-(+3)+(-5)-(-1)$ (6) $(-2)+(+9)-(+1)-(-4)$

Q 3 Express using the addition sign + and parentheses.

(1) $6-8$ (2) $-14-13$ (3) $-4+9-7$ (4) $7-8+6-2$

We can calculate the lined up terms of a math expression using the commutative property and associative property as follows. The positive sign + in the answer can also be removed.

$$\begin{aligned} &2-4+6-1 \\ =&2+6-4-1 \\ =&8-5 \\ =&3 \end{aligned}$$

$$\begin{aligned} &(+2)+(-4)+(+6)+(-1) \\ =&(+2)+(+6)+(-4)+(-1) \\ =&(+8)+(-5) \\ =&+3 \end{aligned}$$

Let me ask!

Is the "−" in "6−8" the subtraction sign, or the negative sign?

P.33

Q 4 Calculate Q2 and Q3.

Ex. 1 Calculate $7+(-8)-5-(-4)$.

Method For calculating a math expression with parentheses, addition and subtraction signs, first line up the terms.

Solution

$$\begin{aligned} &7+(-8)-5-(-4) \\ =&7-8-5+4 \\ =&7+4-8-5 \\ =&11-13 \\ =&-2 \end{aligned}$$

Answer -2

$$\begin{aligned} &7+(-8)-5-(-4) \\ =&7+(-8)-5+(+4) \\ =&7-8-5+4 \end{aligned}$$

Make sure to explain how you found your answer.

Q 5 Calculate.

(1) $-3+(-2)-(-9)$ (2) $8-(+7)-5$

(3) $-2-(-3)+7+(-4)$ (4) $3+(-8)-(-5)-1$

Q 6 Calculate.

(1) $11-17+13$ (2) $-14+19+12-20$

(3) $-3.1-5.9$ (4) $-0.6-(-1)$

(5) $\frac{1}{6}-\frac{3}{4}$ (6) $-\frac{2}{7}+\frac{6}{7}-\frac{3}{7}$

Try it out
P.35
Enhancement 1-3

We can now solve problems by changing the addition and subtraction of positive and negative numbers into a form with lined up terms.

Can we do the multiplication and division of positive and negative numbers in the same way as addition and subtraction?
P.36, 43

Close up

The Meaning and Calculation of "6 − 8"

In elementary school, we did not cover subtraction which takes a larger number from a smaller one such as "6 minus 8". By using negative numbers as well as positive numbers and 0, we are now able to do these kinds of subtraction.
We can see "6−8" as "6 minus 8", but as we learned on page 31, we can also see it as lined up terms of "6 plus −8".

subtraction

$6-8$ ··· Add the positive sign + to 8

$=6-(+8)$ ··· Change the subtraction into an addition

$=6+(-8)$ ··· Line up the terms

minus

$=6-8$

So we can consider the "−" in "6−8" both as a subtraction sign as well as a negative number sign.

Let's Check

1 Addition
[P.23] Ex.3 Ex.4 Q 6

Calculate.

(1) $(+3)+(-2)$ (2) $(-4)+(-6)$

(3) $(-14)+(+5)$ (4) $(-8)+(+8)$

2 Subtraction
[P.28] Ex.3
[P.29] Q 5

Calculate.

(1) $(+2)-(+9)$ (2) $(+1)-(-5)$

(3) $(-6)-(-17)$ (4) $0-(-12)$

3 Calculation with Both Addition and Subtraction
[P.32] Q 4 Ex.1

Calculate.

(1) $(+5)+(-18)+(-5)$ (2) $(-9)-(-8)+(-4)$

(3) $2-7$ (4) $-4-5$

(5) $-2+10-5$ (6) $3-7-4+8$

(7) $16-(+17)-13$ (8) $(-3)+6+(-7)-(-9)$

Close up

Does the Commutative Property and Associative Property Hold True for Subtraction?

On page 25, for addition of positive and negative numbers, we learned that

Commutative property $a+b=b+a$

Associative property $(a+b)+c=a+(b+c)$

hold true.

Do they also hold true for subtraction? Let's compare the following.

(1) ⓐ $(+2)-(+3)$ ⓑ $(+3)-(+2)$

(2) ⓐ $\{(+2)-(+3)\}-(+5)$ ⓑ $(+2)-\{(+3)-(+5)\}$

(1) and (2) have different ⓐ and ⓑ, respectively. We can see that commutative property does not hold true with (1) ⓐ and ⓑ, and associative property does not hold true with (2) ⓐ and ⓑ. Therefore, commutative and associative properties cannot be used for subtraction. However, by changing subtraction into an addition-only math expression, both commutative property and associative property can be used.

Enhancement 1

→ Addition and Subtraction

Let's use what we have learned for self-study and calculation practice.

1 Addition

(1) $(+11)+(+4)$

(2) $(-6)+(-12)$

(3) $(+8)+(-1)$

(4) $(+3)+(-10)$

(5) $(+16)+(-16)$

(6) $(-7)+(+2)$

(7) $(-9)+(+13)$

(8) $(+0.6)+(-1.8)$

(9) $(-2.7)+(-3.5)$

(10) $\left(-\frac{1}{3}\right)+\left(+\frac{1}{2}\right)$

(11) $\left(-\frac{3}{4}\right)+\left(-\frac{5}{12}\right)$

2 Subtraction

(1) $(+8)-(+4)$

(2) $(+3)-(+9)$

(3) $(+5)-(-2)$

(4) $0-(-13)$

(5) $(-7)-(+2)$

(6) $(-9)-(-1)$

(7) $(-2)-(-15)$

(8) $(-1.9)-(+1.4)$

(9) $\left(+\frac{1}{6}\right)-\left(-\frac{1}{2}\right)$

(10) $\left(-\frac{2}{7}\right)-\left(+\frac{5}{14}\right)$

3 Calculation with Both Addition and Subtraction

(1) $(-3)+(+2)-(+5)$

(2) $(+6)-(-7)+(-13)$

(3) $(-6)-(+1)+(-3)-(-8)$

(4) $3-8$

(5) $-6+9$

(6) $-7-4$

(7) $-18+18$

(8) $5-19$

(9) $-2+6-8$

(10) $7-9-5$

(11) $4-7+10-1$

(12) $-12+4-3+7$

(13) $0.4-1.9$

(14) $-1.3+2.7$

(15) $-\frac{2}{5}-\frac{3}{5}$

(16) $\frac{4}{9}-\frac{5}{6}$

(17) $-2+(-10)-6$

(18) $13+(-2)-5-(-7)$

(19) $-7-(+8)-(-3)+9$

(20) $1+(-0.6)-0.8$

(21) $-\frac{1}{3}+\frac{1}{6}-\left(-\frac{2}{3}\right)$

Answers on P.285